Optimization of matrix gradient coil switching for a limited number of amplifiers

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Target Audience: Researchers interested in matrix gradient coil systems.

Purpose: Matrix coils have recently been introduced for shimming as well as encoding[1-4]. In order to drive each coil element independently, each element requires its own amplifier, which comes at great expense. To reduce costs, a switching circuit can be used to drive clusters of coil elements simultaneously with a reduced number of amplifiers[5]. This work focuses on algorithmic approaches for finding suitable coil element clusters that are able to create an approximation to a desired target field. This resembles a combinatorial problem related to the multiple knapsack problem[6] and is NP-hard. The large amount of possible combinations prohibits an exhaustive search, thus heuristics are used instead to find good solutions fast. We use Simulated Annealing (SA)[7] to directly solve the combinatorial problem and introduce two more algorithms that are based on clustering elements with similar currents. Each cluster can hold a limited number of coil elements, which corresponds to inductance limits of the amplifiers.

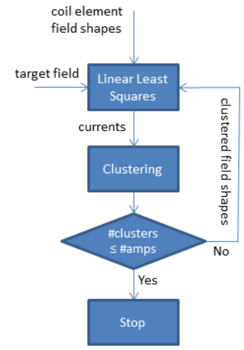

Figure 1: Flow chart for pairwise and sequential clustering

Methods: The coil element field shapes were simulated based on a cylindrical matrix gradient coil design with 84 elements distributed on 7 rings with 12 elements each. A cylindrical volume covering the entire space inside of the coil was used as the volume of interest. The target fields investigated were the first 20 basis fields computed with the SVD on the element fields[8-9]. These fields are orthogonal to each other and can be achieved with zero error by independently driven coil elements. The resolution of both the field maps and target fields is 32x32x80. For SA we provide a parameter vector filled with all coil indices, where the position of an index within the vector defines which cluster/amplifier it is assigned to. The annealing step swaps elements between clusters with the number of swaps per iteration depending on the current temperature. The cost function is defined as the l2-norm of the difference between the target field and the fitted (clustered) field. However, due to the length of the parameter vector and the difficulty of solving combinatorial problems, faster methods than SA are desirable. The second algorithm, pairwise clustering, starts with independently driven coil elements and reduces the number of required amplifiers by a factor of two in each iteration. Each iteration involves a fit of the given field shapes to the target field in the least squares sense in order to obtain the current for each coil element, which are then clustered into pairs of two based on the similarity of their currents. The next iteration proceeds similarly but with the clustered elements instead of the individual elements. This procedure is repeated until the number of clusters is lower or equal to the number of available amplifiers. The third algorithm, sequential clustering, follows the same principle apart from the clustering step, which only clusters two elements/clusters at a time. This increases the computational load as more fitting procedures are necessary, but allows stopping the algorithm whenever the available number of amplifiers is reached and promises better clustering. The principle of these algorithms is depicted in Fig. 1. For this simulation study it was assumed that the maximum number of elements per cluster is 40. All algorithms were implemented in MATLAB (The Mathworks, USA) using the Optimization Toolbox.

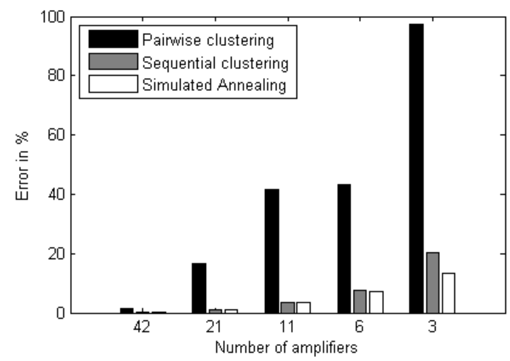

Figure 2: Error for all algorithms and different numbers of amplifiers.

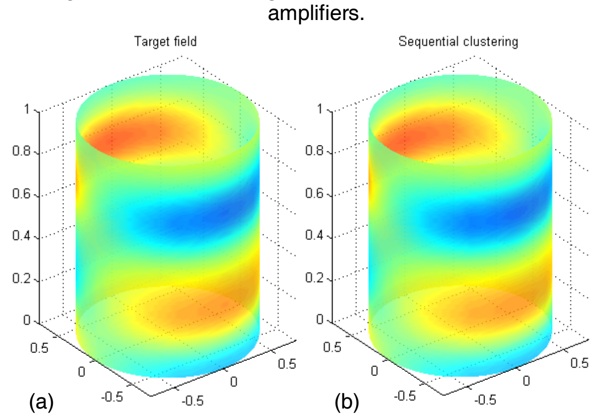

Figure 3: Illustration of the field distribution on the surface of the volume of interest for the target field (a) and the result of sequential clustering (11 clusters) (b).

Results & Discussion: Twenty target fields were investigated, but due to space restrictions only one representative example is shown in Fig. 2 and 3. The remaining target fields showed a similar behavior. Figure 2 shows the normalized error $err = \|x - target\|/\|target\|$ (where $target$ is the target field, x is the computed field after clustering and $\|\cdot\|$ is the l2-norm) for each clustering technique depending on the number of amplifiers. The investigated numbers of amplifiers were 42, 21, 11, 6 and 3. For a large number of amplifiers, all methods perform well; however, many iterations and tedious temperature tuning are necessary for SA. For a lower number of amplifiers the methods start to diverge with pairwise clustering performing the worst with an error approaching 100% for three amplifiers. Sequential clustering shows an overall good performance, but may fail when it is not possible to further reduce the number of clusters without violating the hardware constraints. SA does not suffer from this problem but requires very long computation times. However, SA can be sped up by using the result of pairwise or sequential clustering as a starting point. In this case, 10-20 minutes are sufficient to considerably reduce the error for a low number of amplifiers. Pairwise and sequential clustering need approximately 1s and 15s, respectively. For an indicative number of amplifiers of 11, the error for SA and sequential clustering is always below 4% (mean error 1.5% and 1.7%, respectively) and for pairwise clustering below 50% (mean error 15.75%) for all investigated target fields. As expected, we observed an approximately linear decrease of the coil efficiency with the number of amplifiers down to 11 amplifiers. For fewer amplifiers, coil efficiency decreases significantly faster. Figure 3 illustrates the similarity of the result of sequential clustering (11 amplifiers) to the desired target field. In future, additional hardware constraints given by the amplifiers and switching circuit can be incorporated into the presented algorithms as well as constraints on the coil efficiency.

Conclusion: This work shows that fewer amplifiers than coil elements do not necessarily reduce the flexibility of a matrix gradient coil, as long as the switching circuit suitably assigns the coil elements to amplifiers.

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