Novel Approaches in the Coupled Circuit Simulation of Eddy Currents Induced by Cylindrical Gradient Coils

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Target Audience: MR Engineers interested in modeling eddy current effects by simulation.

Purpose: The purpose of this study is to analyze temporal and spatial responses of eddy currents induced by Z-gradient and X-gradient coils in a 9.4 T narrow bore (54 mm i.d.) MRI system dedicated for microscopic study by implementing novel approaches in coupled circuit numerical method. Previously we implemented this method for planar coil[1] by using solid angle form of Ampere's law[2]. In this study, we extended this approach for cylindrical coil. Novel calculation and modeling approaches were implemented considering cylindrical coils and cryostat bore.

Methods: We consider innermost three Cu (C1220, resistivity $2.0284\times10^{-8}\,\Omega$.m) bore as they are closer to gradient coil assembly. Bore dimensions are given in Table1. We divided each bore into thin cylindrical sublayers and, each sublayer into ring-shaped subdomains along Z-axis for Gz coil and bar-shaped subdomains along azimuthal direction for Gx coil. Simulation parameters are given in Table1 and modeling approach is illustrated in Fig.1. Coupled equation is expressed as[3]: $\mathbf{M}_{ii}\frac{d\mathbf{I}(t)}{dt}+\mathbf{R}_i\mathbf{I}(t)=-\mathbf{M}_{is}\frac{di_s(t)}{dt}$, where $\mathbf{M}_{ii}$ is matrix of self-inductance and inductive coupling of subdomains, $\mathbf{R}_i$ is resistance matrix and $\mathbf{M}_{is}$ is matrix of inductive couplings between coil and subdomains. $\mathbf{M}_{ii}$ were computed by formulas[4] in Table2 (column 2 and 3); $\mathbf{M}_{is}$ by solid angle form of Ampere's law[1,2].

To consider inductive disturbances of neighboring subdomains, we followed actuance calculation suggested by P. R. Vein[5] (illustration in Fig.1) – equation in Table3. Magnetic field at boundary undergoes attenuation inside materials due to induced antiparallel magnetic moments[6]. For subdomains inside the material (e.g. C_2 in Fig. 1), we also considered this effect. The inductive coupling becomes: ((magnetic flux linkage in vacuum)+(Flux inside material)-(N_{is})). We solved the coupled differential equation implementing Eigen method approach explained in[2]. For verification gradient echo shift eddy current measurement method[7] was followed.

Table3 Inductive disturbances:

$$N_{is}=\frac{1}{\Delta}\sum_{p=1}^{n}\sum_{q=1}^{n}H_{1p}H_{2q}\Delta_{pq},$$

M_{is} inductive coupling between C_1 and C_2
H_{1p} inductive coupling between C_1 and D_p
H_{2q} inductive coupling between C_2 and D_p
m_{pq} inductive coupling between D_p and D_q
m_{pp} self-inductance of D_p
R_p resistance of D_p
n the number of neighboring subdomains D_p

$$\Delta=\begin{vmatrix}\frac{R_1}{j\omega}+L_1 & m_{12} & \vdots & m_{1n}\\ m_{21} & \frac{R_2}{j\omega}+L_2 & \vdots & m_{2n}\\ & \cdots & \vdots & \cdots\\ m_{n1} & m_{n2} & \vdots & \frac{R_n}{j\omega}+L_n\end{vmatrix}$$

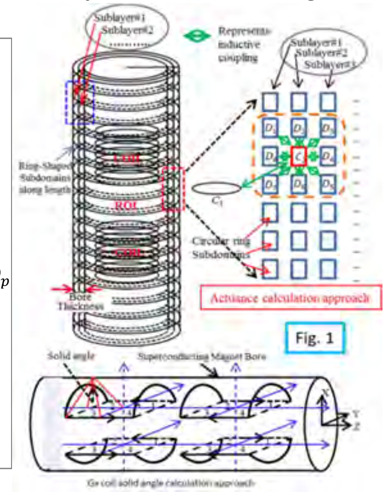

Table1	Dimensions			Simulation parameters				
	Thickness	Length	Diameter	Sublayer thickness	Sublayers	Subdomain/sublayer (Gz)	Subdomain/ sublayer (Gx)	Subdomain width
Bore1	1.63 mm	684 mm	53.84 mm	.0676 mm	24	400	172	1 mm
Bore2	1.63 mm	797 mm	61.14 mm	.0676 mm	24	400	196	1 mm
Bore3	1.63 mm	971 mm	69.74 mm	.0676 mm	24	400	223	1 mm

Table2	Self-inductance	Inductive coupling (subdomain-subdomain)	Solid angle (gradient-subdomain)
Gz	$\mu_o a\left[\log\frac{8a}{b}-\frac{1}{2}+\frac{b^2}{32a^2}\left(\log\frac{8a}{b}+\frac{1}{4}\right)\right]$ a is ring radius, b is width	$M_{ii}=2\mu_o\sqrt{\frac{a_1a_2}{k}}\left(\left(1-\frac{k}{2}EllipticK(k)\right)-EllipticE(k)\right)$, $k=\frac{4a_1a_2}{((z_1-z_2)^2+(a_1+a_2)^2)}$, a_1, a_2 radius; z_1, z_2 positions along Z-axis of two rings	$2\pi\left(\frac{z}{\sqrt{z^2+x^2+y^2}}-\frac{z}{\sqrt{z^2+x^2+y^2+r^2}}\right)$, r is radius of a circular loop[1]
Gx	$\frac{\mu_0}{2\pi}l\left[Log\left(\frac{2l}{0.2235(g+c)}\right)-1+\frac{0.2235(g+c)}{l}\right]$ l, g, c are length, thickness and width of bar	$\frac{\mu_0}{2\pi}l\left[Log\left(\frac{l}{d}+\sqrt{1+\frac{l^2}{d^2}}\right)-\sqrt{1+\frac{d^2}{l^2}}+\frac{d}{l}\right]$, l, d are length, geometric mean distance	$\tan^{-1}\frac{(z_1-z_p)(y_1-y_p)}{x_p\left[(z_1-z_p)^2+(y_1-y_p)^2+x_p\right]^{\frac{1}{2}}}$ p is field point[8]

Results and discussion: Details of the simulation results are given in Fig. 2 and Fig. 3. The light green shaded region corresponds to approximate position of gradient coils. For both coil, similar but opposite amplitude response are seen due to reverse directional gradient current flow. Time constants increases with increasing diameter of Cu bore (Fig.2,3(b)). Total gradient eddy field simulated for inner three bore are compared with the experimental results - Fig.2(d) for Gz and Fig.3(d) for Gx coil. A good agreement was found between simulation and experiment. It is seen that inner bore (Bore1) generates the most eddy fields (Fig.2,3(c)). The implementation of solid angle calculation made it possible to calculate electric induction of a loop of coil as a whole or by taking segments that makes it more analytic and speed up calculation processes.

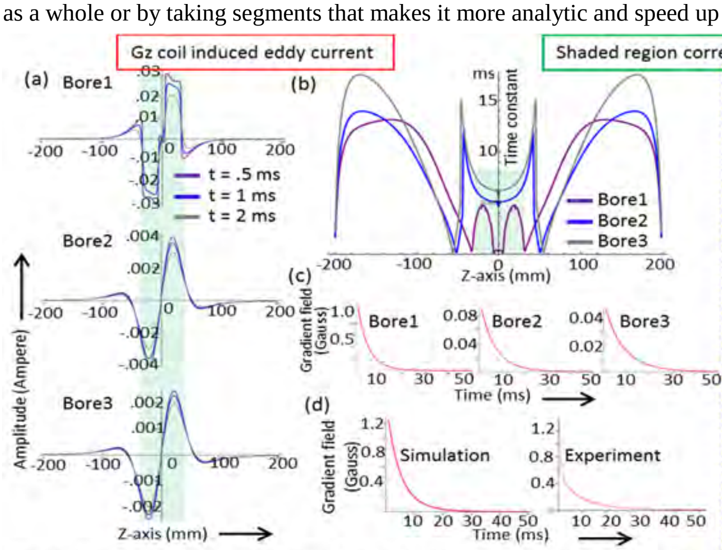

Fig. 2. Simulated results for innermost 3 bore: Eddy current (a) amplitudes and (b) time constants; (c) gradient eddy field . (d) Total gradient field – comparative study.

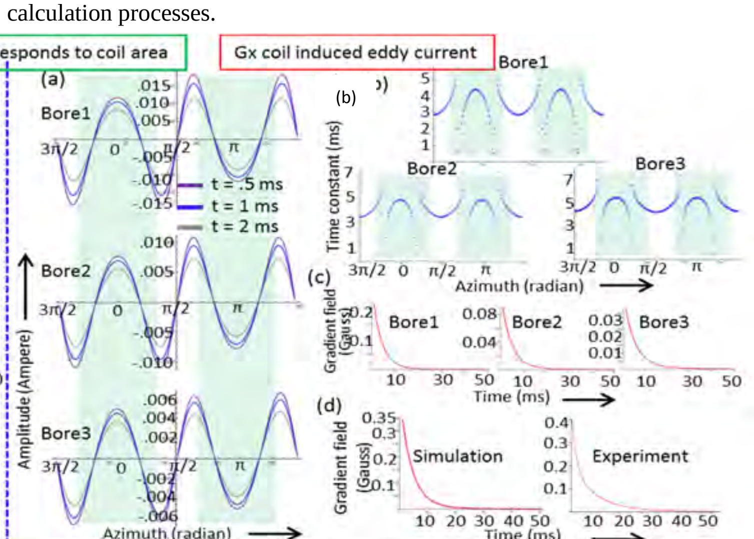

Fig. 3. Simulated results for innermost 3 bore: Eddy current (a) amplitudes and (b) time constants; (c) gradient eddy field . (d) Total gradient field – comparative study.

References: [1] M. S. H. Akram et al, J. Mag. Reson., 245 (2014) 1-11. [2] W.T. Scott, *The phys Elec Magn*, 2nd Ed, John Wiley& Sons, 1959. [3] M.J. Sablik, et al, IEEE Trans. Mag. 20 (1984) 500 – 506. [4] E. B. Rosa, *The self and mutual inductances of linear conductors,* National Bureau of Standards, Vol. 4, (1908). [5] P. R. Vein, Int. J. Electron. 13 (1962) 471 – 495. [6] R. Plonsey, R.E. Collin, *Princ. Applic Electromag Fields*, McGraw-Hill, 1961. [7] V. J. Schmithorst et al, Magn. Reson. Med., 47 (2002) 208 – 212. [8] H. Gotoh, et al, Nucl. Inst. Meth. 96 (1971) 485–486.