Lorentz cavity field in media with magnetic structure

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PURPOSE: To clarify the dependence of the Lorentz cavity on the structure of investigated tissue on the cellular level.

INTRODUCTION: Although the notion of the Lorentz cavity[1,2] belongs to the fundamentals of NMR, it has recently become a subject of controversy[3,4]. The central problem is the value of the mean magnetic field experienced by the NMR-visible spins and reported to the receiving coils. The extension of the original Lorentz idea to NMR implies that the fast motion of the spin-bearing molecules results in an effective averaging to zero of the dipole fields contributed by the molecules in a vicinity of a given spin (**Fig. 1**). He and Yablonskiy[5] argued that a similar averaging takes place in biological tissues when considering the field induced by the magnetic susceptibility variations between different cell species. In particular, myelinated axons in white matter result in a Lorentz cavity in the form of a cylinder, which implies a specific anisotropy of the induced field[5,6] and its sensitivity to the axon integrity[7].
Here we point out that the inclusion of cells in the Larmor cavity crucially depends on the relation of their size to the diffusion length during the acquisition of the free induction decay (FID). Aiming at experiments with microbead suspensions in water we demonstrate in numerical simulations how the mean magnetic field interpolates between the Lorentz sphere in agreement with Ref. (2) for small beads and the classical one for large beads (**Fig. 2**). In the former case the Lorentz cavity includes a large number of microbeads and the subtracted field is determined by the bulk magnetic susceptibility of suspension (water + beads). In the latter case the Lorentz cavity includes only water, while the effect of beads is to be calculated in a more detailed way.

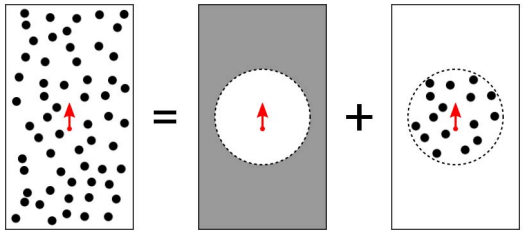

Fig. 1: The origin of the Lorentz cavity. Particles inducing a dipole field (black) can be separated in a far and a near region around an NMR-visible spin (red). The far region contributes a field according to its averaged (bulk) magnetic susceptibility (gray). The field in the near region is effectively averaged to zero by motion when the particles are small (molecules and small objects). For large objects the averaging is incomplete. The present simulations address the value of the mean field in the near region (the right image).

METHODS: Monte Carlo Simulations of spin diffusion were performed in three-dimensional media of randomly placed non-overlapping impermeable spheres occupying various volume fractions between 1% and 35%, where the sphere radius ρ was kept the same for all media. Variable parameters were the diffusion coefficient D and the strength of field variations δΩ induced by the spheres. The recorded FID signals were cut at noise level and zero filled to obtain a reasonable resolution in frequency space. The mean frequency was determined as the maximum of the smoothed spectral line with the error given by the spectral resolution.

RESULTS demonstrate the anticipated interpolation of the frequency shift between the vanishing mean field for small beads and an incomplete one for large beads (**Fig. 2**). The interpolation is not monotonous: a small diffusivity results in a stronger effect than the zero one. This can be understood in terms of a correlation between the local Larmor frequency and its gradient (data not shown). This effect is expected to be dependent on the shape of field-inducing particles. Further increase in the diffusivity results in the approach to the limit of vanishing mean field. Result presentation is made in terms of dimensionless quantities according to the scaling, which is inherent to the system. In reality, the variable parameters are the bead size and their magnetic susceptibility rather than the diffusivity.

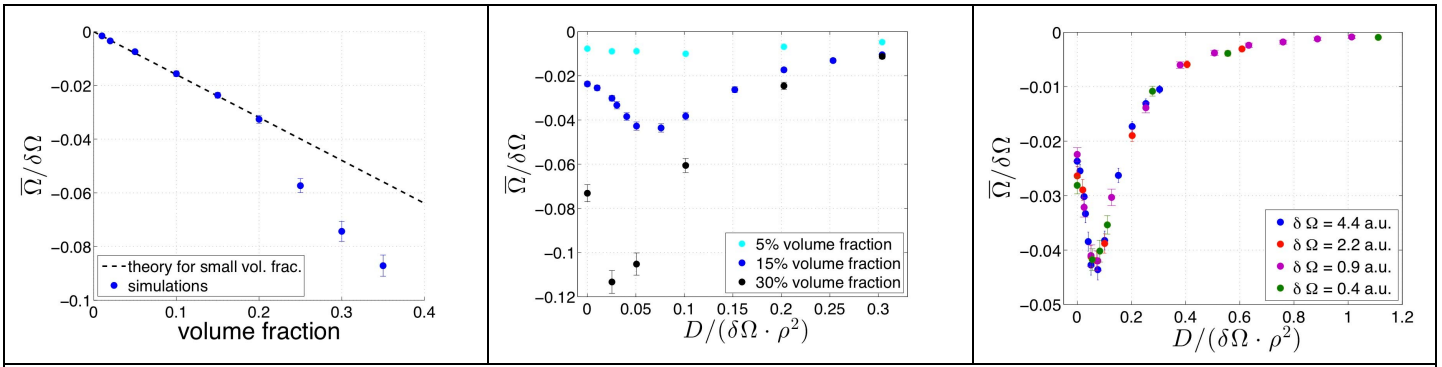

Fig. 2: Simulation results. Left: The mean Larmor frequency shift for non-diffusing spherical particles in comparison with the theoretical prediction[8] for low volume fractions. Middle: Dependence of the frequency shift on the dimensionless diffusivity for different volume fractions of particles. Right: The same dependence for the 15% volume fraction for different particle magnetizations. Four lines collapse to a single one thus demonstrating the scaling properties of the results.

DISCUSSION: Our results support the conjecture by He and Yablonskiy[5] about the extension of the Lorentz cavity on the field-inducing structural units in the case when such units are small. In this case the mean magnetic field reported by spins in the form of the central spectral line frequency is fully determined by the bulk magnetic susceptibility of the medium obtainable as the weighted mean of all components. This mean value defines both the demagnetizing field and the Larmor cavity field. For larger structural units, the mean field cannot be obtained in such a simple way. The Lorentz cavity is determined by the solvent magnetic susceptibility, while the effect of the structural units should be calculated via a proper solution of the Bloch-Torrey equation. Extending this result to the myelinated axons, we speculate using the plausible parameters (transverse D ~ 0.4 mm^2/ms in extraaxonal space, T_2 ~ 80 ms) that the diffusion length $(4Dt)^{1/2}$ ~ 11 mm is large enough for the majority of axons to account for their susceptibility effect according to Ref. (3). However, in areas with larger axons the accuracy of this approximation may become insufficient.

Acknowledgement: This work was supported by DFG grant KI-1089/6-1 **References:** (1) H.A. Lorentz. The theory of electrons. B.G. Teubners, 1909; (2) S.C. Chu et al. MRM 13 (1990) 239; (3) D.A. Yablonskiy et al. MRM 72 (2014) 4; (4) J.H. Duyn, T. Barbara. MRM 72 (2014) 1; (5) X. He, D.A. Yablonskiy. PNAS 106 (2009) 13558; (6) J. Luo et al. MRM 71 (2014) 1251 (7) D.A. Yablonskiy et al. PNAS 109 (2012) 14212 (8) D.A. Yablonskiy, E.M. Haacke. MRM 32 (1994) 749