Optimal Motion Encoding Scheme for MR Elastography

Temel Kaya Yasar[1], Yifei Liu[2], Dieter Klatt[3], Richard L Magin[3], and Thomas J Royston[3]

[1]*Radiology, Icahn School of Medicine at Mount Sinai, New York, New York, NY, United States,* [2]*Mechanical Engineering Department, University of Illinois at Chicago, Chicago, IL, United States,* [3]*Biomedical Engineering Department, University of Illinois at Chicago, Chicago, IL, United States*

Target Audience: Clinical scientists and physicists interested in MR Elastography

Table 1ULTIMATe MRE notation

f_m	Mechanical Motion Frequency
f_g	MEG Frequency
N	Number of Time Steps (NoTS)
VTD	Virtual Time Domain
VFD	Virtual Frequency Domain
t	Time domain index
n	VTD index
f	Frequency domain index
α	VFD index
g	Gradient shape vector field
$\mathbf{g}_{mv}$	m^{th} frequency, v^{th} direction of **g**
u	Mechanical motion vector field
$\mathbf{u}_m$	m^{th} frequency component of **u**
γ	gyromagnetic ratio
M	Total number of mechanical motion at different frequencies
m	Mechanical frequency index
V_m	number of motion directions to be encoded at m^{th} frequency
v	Index for direction

Introduction: Magnetic Resonance Elastography (MRE) is a non-invasive imaging method that enables measurement of mechanical parameters of the subject under investigation[1]. Harmonic waves are introduced into the subject via external actuators and the three dimensional wave propagation patterns are fully or partially captured using a phase contrast imaging method. This wave propagation information is then processed into mechanical property maps, which are called stiffness maps. MRE can be implemented into most of the standard imaging sequences by embedding motion encoding gradient (MEG) functions into the sequence. The same scan is repeated for shifted MEGs to extract the information of wave propagation direction. There are methods to reduce the number of repetitions[2,3]; however, these are at the cost of lower SNR. Recently introduced two new methods, SDP[4] and SLIM[5], enables encoding of more information for a given acquisition time without sacrificing SNR or motion encoding efficiency. **Problem:** The acquisition of motion displacement in multiple directions or at multiple frequencies are conventionally conducted in one direction or using one frequency at a time. However, this approach prolongs the data acquisition, which results in image mis-registration, artifacts due to patient motion and physiological changes, and/or discomfort to the patient due to increased number of breath holds. SDP and SLIM provide optimal solutions to this problem, they do not provide the flexibility of picking a particular frequency in a particular direction, such as encoding three different frequencies in the same direction. **Objective:** We introduce a generalized optimal motion-encoding scheme for MRE. The sample interval modulation technique of the SLIM algorithm is combined with the selective spectral displacement technique of the SDP algorithm into a generalized method that is called Unified SampLing Time Interval ModulaTion (ULTIMATe MRE). In other words, ULTIMATe MRE makes it possible to encode mechanical motion of *m* frequencies into V_m directions without sacrificing SNR given sufficient gradient power and displacement amplitude. Since the existing notation of MRE in literature is ambiguous and insufficient for ULTIMATe MRE, here we introduce a new mathematical structure and notation. **Theory:** This new set of notation is created to clarify the 'frequency' and 'time' terms in different domains (Table 1), and distinguish the frequency of mechanical motion and MEG shape (An MEG function is created to encode single motion information, which is a wave image (single slice, multi slice or volumetric) obtained by encoding mechanical motion at only one frequency and in one direction). Considering multiple mechanical frequencies and multiple gradient shapes encoding multiple directions for each frequency component, the conventional phase accumulation formula (1) can be extended as a summation (2a) of individual phase accumulation components (2b), where s_{mv} is the delay between mechanical motion of the m^{th} frequency and the gradient shape of the m^{th} frequency and the v^{th} direction for the n^{th} time step. In practice we can only measure $\phi_{mv}(s)$ for a finite number of *s* and from a mathematical point of view this process is no different from the sampling of an analog signal. Therefore, MRE wave images across different time offsets (in other words different *s* values) are actually time samples of wave propagation. If we denote the sampling interval for the v^{th} direction of the m^{th} mechanical frequency as ΔT_{mv}, the sampled phase accumulation can be written as in (3), where $\phi_{mv}[n]$ is the phase accumulation due to gradient shape $\mathbf{g}_{mv}$. In general, the timing of the sampling operation can be controlled for each motion information individually since ΔT_{mv} is user adjustable. This allows us to acquire and arrange sampled time instances of each motion information independently. Since sampling time instances do not need to be consecutive and equally spaced in time, we will name this discrete domain (*n*) as the Virtual Time Domain (VTD) to avoid ambiguity with the actual time domain (t). In order to extract all the motion information, the sampled phase accumulation should be analyzed in the frequency domain. The discrete frequency domain is named as the 'virtual frequency domain' (VFD) because ΔT_{mv} is multiplied by actual frequency f_m in (4) and this changes the location of that information in the virtual frequency domain arbitrarily. In order to obtain the VFD, a discrete Fourier transform is applied to the sampled phase accumulation in VTD. The sampled form of general phase accumulation is given in (5). The phase accumulation created by individual gradient shapes is the integration of the projection of mechanical displacements on that particular gradient vector multiplied by gyromagnetic ratio as given in (6). The gradient shape, $\boldsymbol{g}_{mv}(n)$, non-zero in the interval $s_{mv(n)} < t < s_{mv(n)} + k/f_m, k \in \mathbb{Z}^+$, is given in (7) as a function of starting time $s_{mv}(n)$ and rewritten in (8) as a function of sampling interval ΔT_{mv}. In general, if a mono-frequency function *h(t)* is sampled at $b2\pi n/N$ phase instances, the signal will fall into the b^{th} frequency bin in the discrete Fourier domain (9). Using this property, ΔT_{mv} is solved for desired frequency bin placement α as in (10). This concludes the derivation of each gradient shape, $\boldsymbol{g}_{mv}(n)$, which will place the v^{th} direction component of $\mathbf{u}_m$ in the α^{th} frequency bin.

$$\phi(s) = \gamma \int_s^{s+T} \boldsymbol{g}(t - s(n)).\boldsymbol{u}(t) \quad (1)$$

$$\phi = \sum_{m=1}^{M} \sum_{v=1}^{V_m} \phi_{mv} \quad (2a)$$

$$\phi_{mv} = \Phi_{mv} \cos(2\pi f_m s_{mv}(n) + \theta_{mv}) \quad (2b)$$

$$\phi_{mv}[n] = \phi_{mv}(n\,\Delta T_{mv}) \quad (3)$$

$$\phi_{mv}[n] = \Phi_{mv} \cos(2\pi f_m (n\Delta T_{mv}) + \theta) \quad (4)$$

$$\phi[n] = \sum_{m=1}^{M} \sum_{v=1}^{V_m} \phi_{mv}[n] \quad (5)$$

$$\phi_{mv}[n] = \gamma \int \boldsymbol{g}_{mv}(n,t).\boldsymbol{u}_m(t) \quad (6)$$

$$\boldsymbol{g}_{mv}(n) = G_{mv} \sin(2\pi f_m (t - s_{mv}(n)) \hat{e}_{mv} \quad (7)$$

$$\boldsymbol{g}_{mv}(n) = G_{mv} \sin(2\pi f_m (t - (n-1)\Delta T_{mv}) \hat{e}_{mv} \quad (8)$$

$$h[n] = e^{\frac{j2\pi n}{N}\alpha}, n \in \{0, \dots, N-1\} \quad (9)$$

$$\Delta T_{mv} = \alpha/(N f_m) \quad (10)$$

Table 2 Coefficient of variation (CV) between ULTIMATe MRE and Standard MRE shear stiffness mean values on 4 different beads. Last column is mean of CV.

Bead	#1	#2	#3	#4	Mean
5 kHz	0.18%	1.26%	0.40%	0.20%	0.51%
6 kHz	0.38%	1.39%	0.81%	0.20%	0.69%
7 kHz	0.93%	1.21%	0.00%	0.20%	0.58%

Figure 1 4 Beads

Methods: ULTIMATe-MRE was applied to a phantom consisting of randomly placed six agarose beads (1.08% w/w) embedded in agarose gel (1.3% w/w). The basic MRE setup used in the experimental 11.7T Bruker vertical MRI system has been described before[4]. We used a gradient-echo based ULTIMATe-MRE pulse for data acquisition in 8 axial slices covering all the beads with the following sequence parameters: TR=150 ms; TE=2.94 ms + T_{MEG}; flip angle=30°, FOV=10x10 mm; matrix size=128 x 128; slice thickness=1 mm; MEG amplitude=80 G/cm, NoTS = 8. As shown in a previous study[4], the excitation signal was a superposition of 5 kHz-, 6 kHz- and 7 kHz-sinusoidal waveforms. MEG gradient cycle numbers were 5, 6 and 7, and the power of the slice gradient distribution was as 20%, 25% and 35% for 5 kHz, 6 kHz and 7 kHz MEG shapes, respectively, which were all oriented in the slice-direction. This case was chosen specifically to demonstrate the unique capability of the ULTIMATEe sequence. The 2D LFE algorithm was applied to the wave images to estimate shear stiffness and the resulting map was spatially averaged over each bead indicated in the magnitude image in Figure 1. For comparison, conventional MRE was executed separately for all three MEG shapes. **Results:**. Shear stiffness maps (Figure 2) are virtually identical. CV between methods was calculated for each bead and each frequency, Table 1. Average CV value is less than 0.7% for all three frequencies. **Discussion and Conclusion:** Excellent agreement of shear stiffness estimation is observed between standard MRE and ULTIMATe MRE. Detailed visual inspection yields slightly smoother stiffness maps for standard MRE. This is due to anticipated lower SNR of ULTIMATe-MRE originating from higher diffusion b values.

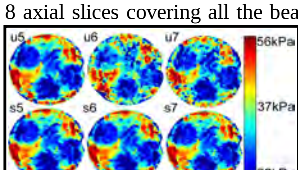

*Figure 2 Shear stiffness maps. **u**: ULTIMATe MRE, **s**: Standard MRE, **#**: frequency in kHz*

References:1- Muthupillai, R., et al. *Science* 269.5232 (1995): 1854-1857. 2- Wang, Huifang, et al. *Phys Med Biol* 53.8 (2008): 2181. 3- Rump, Jens, et al. *MRM* 57.2 (2007): 388-395. 4- Yasar, Temel K., et al. *Phys Med Biol* 58.16 (2013): 5771. 5- Klatt, Dieter, et al. " *Phys Med Biol* 58.24 (2013): 8663.