Noise map estimation in diffusion MRI using Random Matrix Theory

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TARGET AUDIENCE: Researchers interested in diffusion MRI (dMRI) processing and microstructural modeling.

PURPOSE: **To estimate a spatially varying noise map in dMRI in a model-independent way.** The transition from "weighted" images to full-scale microstructural modeling is increasing the bar on estimating dMRI signal parameters in an unbiased way. As the noise in dMRI is generally non-Gaussian, the need to correct for, e.g., Rician or noncentral χ noise bias, crucially relies on an independent *unbiased estimate for the noise map*. Estimating the noise is challenging as dMRI suffers from low spatial resolution and involuntary motion. Moreover, the noise is generally spatially varying due to the use of parallel imaging techniques. This is the reason why noise estimation has remained a challenging problem, with only a few methods able to deal with the spatially varying nature of the noise. Unfortunately, the existing techniques depend on diffusion model assumptions, or the wavelet transformation to decompose the data in low frequency ("signal") and high frequency ("noise") information. Wavelet-based methods tend to overestimate the noise level, as the high frequency sub-band also contains residual signal due to actual sharp edges in the image. Physiological noise, image misalignment and model inaccuracies, all tend to bias the diffusion model-based methods. **Here we present a novel local noise estimation paradigm based on the random matrix theory (RMT) result for random covariance matrices[1].** It is free of the above limitations and performs better than existing methods. We use principal component analysis (PCA) coupled with RMT, to exploit the redundancy in multi-directional dMRI data.

METHODS: How to separate signal from noise without depending on an often unknown physical model of the signal? We here propose the coupling of a PCA with RMT as an answer. If the noise-free signal is characterized by only a *small number P << N,M* of parameters, compared to the number *N* of diffusion acquisitions and number *M* of voxels, we expect a lot of redundancy in the data – even if both the model and its parameters are unknown. Indeed, PCA based on the singular value decomposition of the *N*×*M* matrix typically shows a few "significant" eigenvalues, and the vast majority of the others originating due to the noise. Unfortunately, distinguishing between the "significant" and "noise" eigenvalues is far from trivial. Remarkably, **while noise in each diffusion direction is random, its contribution to the histogram of covariance matrix eigenvalues becomes deterministic for *N,M* >> 1** (see **Fig. 1**). It is given by the universal Marchenko-Pastur distribution[1]:

$$p(\lambda) = \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{2\pi Q \sigma^2} \quad \text{if} \quad \lambda_- \le \lambda \le \lambda_+ \quad (0 \text{ otherwise})$$

with $\lambda_{+,-} = \sigma^2 \left(1 \pm \sqrt{Q}\right)^2$, σ the Gaussian noise level and $Q = \tilde{N}/M$ ($\tilde{N}$ is the number of *noise-only* components). Distribution fitting, i.e. minimizing the mean squared error between the $p(\lambda)$ above, and the histogram ($n(\lambda)$) of the lowest $\tilde{N}$ eigenvalues by varying σ and $\tilde{N}$, yields an uncorrected estimate of the parameters of interest. To correct for the Rician statistics, the $N - \tilde{N}$ "significant" eigenvalues are used to estimate the underlying *signal*. Both this signal and the *Gaussian* noise level σ return the *Rician* noise level using Koay's inversion technique[2]. We applied this technique within a sliding window (typically *M*=7×7×7). Our technique is compared to wavelet based techniques, proposed by Coupé et al.[3] and Veraart et al.[4] Unlike the former, the latter one also exploits the redundancy of oversampled diffusion MR data to achieve local noise estimation[4].

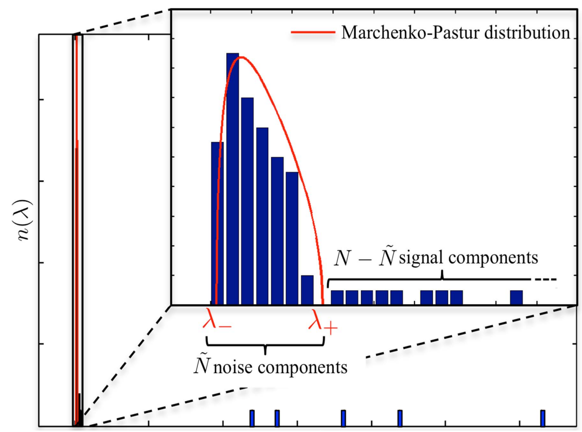

***Fig. 1**: Eigenvalue spectrum of sample covariance matrix of simulated DW data and Marchenko-Pastur distribution superimposed in red.*

RESULTS: *Simulation:* **Fig. 2** shows the noise map estimated from simulated data (average SNR of the non-DW image was 20 and 40). Noise-free (i.e. filtered) DW images (90 gradient directions and b = 1µm²/ms) were derived from Human Connectome Project dMRI data by fitting 4th order spherical harmonics and recomputing directional data; we use this as ground truth. Rician noise with a uniform noise level was artificially added to compare our proposed local RMT-based method (*PCA-RMT*), local wavelet-based noise map estimator[4] (*WAVELET*), and its global counterpart[3]. Both wavelet approaches tend to overestimate the noise; this bias depends on SNR. Moreover, its local version[4] shows structure, whereas our RMT method shows high accuracy and precision regardless of the underlying signal.

Real data: For a dMRI measurement (12 repetitions of b = 0, 60 x b = 1 ms/µm², 12 repetitions of 1 x b = 3 ms/µm²), we applied the proposed method to the b = 1 ms/µm² shell, and compared the result to (I) the wavelet-based local noise map estimator applied on the same shell, and (II) the noise maps derived from the repeated measurements[5] (see **Fig. 3**). The noise map derived from the $b = 0$ repetitions is clearly affected by pulsation artifacts. This effect is not visible in our *bronze* standard based on 12 repetitions from the same b = 3 ms/µm² image. Strong edges (i.e. high frequency information) in the dMRI images shine-through in the wavelet-based approach, whereas our proposed method shows a smooth, artifact-free noise map that is more consistent with the bronze standard than the wavelet-based method (correlation coefficient of 0.9951 and 0.9782, respectively; see scatter plot).

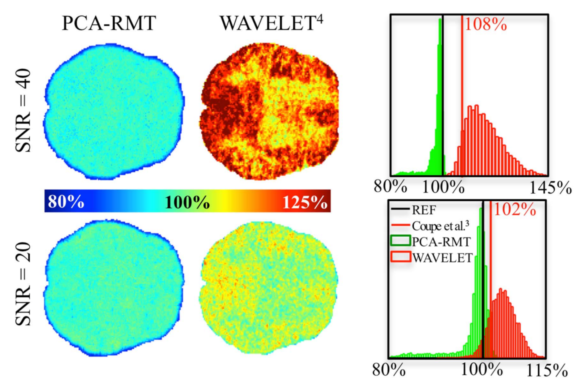

***Fig. 2:** Our proposed method based on Random matrix theory shows high performance, both in terms of accuracy and precision compared to local and global (red line) wavelet-based approaches, regardless the underlying signals.*

DISCUSSION & CONCLUSION: The main novelty of this work is the coupling of the local PCA with the RMT, in order to distinguish between the *noise* and *signal* components *in a model-independent way*, and without the need of an empirically set PCA threshold value[6]. The RMT-PCA outperforms known spatially varying noise estimation methods, and does not require acquisition of repeated measurements, instead utilizing dMRI data redundancy. The results easily generalize onto a noncentral χ distribution[2], and can be applied in other modalities including redundant *time series*, such as fMRI. We will also discuss the closely related RMT-based way to objectively count the number of model parameters *P* (derived from $N - \tilde{N}$), which would be very insightful for microstructural modeling.

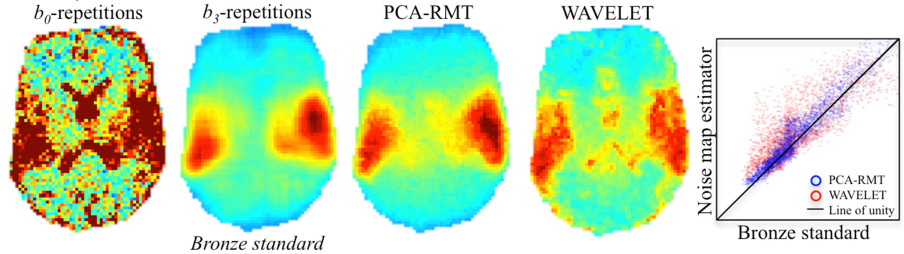

***Fig. 3:** A real data experiments show qualitatively and quantitatively that our proposed method (PCA-RMT) outperforms the wavelet-based method if compared to a bronze standard, which was computed from 12 repeated measurements at high b-value to suppress CSF related physiological noise.*

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REFERENCES: [1]Marchenko et al. (1967) *Mat. Sb. (N.S.)*, 72(114), 507–536 [2]Koay et al. (2006) JMR 179(2):317-22 [3]Coupé et al. (2010) MEDIA, 14(4), 483-93 [4]Veraart et al. (2013) MRM 70(4), 972-84 [5] Maximov et al. (2012) Med Imag Anal 16:536-548 [6]Mangon et al. (2013) Plos One, DOI: 10.1371/journal.pone.0073021