k-t SPARKS: Dynamic Parallel MRI Exploiting Sparse Kalman Smoother

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Introduction: Dynamic parallel magnetic resonance imaging (PMRI) has been widely used in a variety of fast imaging applications to accelerate the data acquisition without apparent compromise of the spatial-temporal resolution. An accurate calibration is the key for successful dynamic PMRI[1-3]. However, the calibration quality typically decreases with both small amount of calibrating signals and motion-induced temporally varying coil sensitivity. In this work, we propose a new, dynamic PMRI exploiting sparse Kalman smoother (k-t SPARKS) for robust calibration and reconstruction in the presence of time-varying coil sensitivity, in which the proposed method incorporates the Kalman smoother calibration and the sparse signal recovery into a single optimization problem, leading to joint estimation of time-varying convolution kernel and full k-space. Simulation and experiments were performed using both the proposed and conventional methods in the free-breathing cardiac cine applications for comparison.

Theory: Under the assumption that the convolution kernel transition between adjacent time frames in dynamic PMRI is slow and thus inter-frame difference remains marginal, the kernel can be described by the following, two linear state-space equations: $\mathbf{g}_{l,t} = \mathbf{g}_{l,t-1} + \mathbf{w}_{l,t}$ **(1)**; $\mathbf{y}_{l,t} = \mathbf{S}_t \mathbf{g}_{l,t} + \mathbf{v}_{l,t}$ **(2)**, where $\mathbf{g}_{l,t}$ is the convolution kernel at the l^{th} coil and the t^{th} time frame, $\mathbf{y}_{l,t}$ is the measured target signals, $\mathbf{S}_t$ is the matrix consisting of source signals, and $\mathbf{w}_{l,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_{l,t})$ and $\mathbf{v}_{l,t} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{l,t})$ represent the process and the measurement noises with normal distribution, respectively. Unlike the Kalman filter[4], in this work all available data (past, current, future observations) in k-t space are included in estimating the kernel by solving the optimization problem in a framework of the fixed-interval Kalman smoother[5]:

$$\mathbf{g}_{l,t}^{KS} = \mathbf{min}_{\mathbf{g}_l^{KS}} \frac{1}{2} \|\mathbf{S}\mathbf{g}_l - \mathbf{y}_l\|_{\mathbf{R}_l^{-1}}^2 + \frac{1}{2}\sum_{t=1}^{T} \|\mathbf{g}_{l,t} - \mathbf{g}_{l,t-1}\|_{\mathbf{Q}_{l,t}^{-1}}^2 \qquad \mathbf{(3)}$$

where $\mathbf{S} = \mathbf{diag}(\mathbf{S}_1, \cdots, \mathbf{S}_T)$, $\mathbf{R}_l = \mathbf{diag}(\mathbf{R}_{l,1}, \cdots, \mathbf{R}_{l,T})$, and $\mathbf{y}_l = \mathbf{vec}([\mathbf{y}_{l,1} \cdots \mathbf{y}_{l,t}])$. Note that this is a non-causal process in that the convolution kernel is estimated using all available observations. Since the unknowns of convolution kernel rapidly grow with increasing number of time frames, the problem becomes computationally intractable. For this reason, the convolution kernel is recursively estimated by exploiting forward-backward algorithm: 1) the Kalman filter in the forward time direction, 2) the information filter (defined as the reverse-time filter with respect to the forward filter) in the backward time direction, and 3) the optimal combination (smoothing) of the two estimates in the forward and backward directions. Incorporating the Kalman smoother calibration step into the constrained optimization problem, we propose a novel dynamic PMRI, leading to joint estimation of time-varying convolution kernel and full k-space directly in k-t space:

$$\mathbf{P0:}\ \mathbf{h}(\mathbf{x}_l, \mathbf{g}_l) = \mathbf{min}_{\mathbf{x}_l, \mathbf{g}_l} \frac{1}{2} \|\mathbf{S}\mathbf{g}_l - \mathbf{y}_l\|_{\mathbf{R}_l^{-1}}^2 + \frac{1}{2}\sum_{t=1}^{T} \|\mathbf{g}_{l,t} - \mathbf{g}_{l,t-1}\|_{\mathbf{Q}_{l,t}^{-1}}^2 + \boldsymbol{\lambda}_1 \|\mathbf{F}_t \mathbf{F}_s \mathbf{x}_l\|_1, \quad \mathbf{s.\ t.}\ \mathbf{x}_l = \mathcal{A}(\mathbf{y}, \mathbf{g}_l) \qquad \mathbf{(4)}$$

where $\mathbf{F}_s$ and $\mathbf{F}_t$ are the Fourier operator in the spatial-temporal direction, respectively, and $\mathcal{A}(\cdot)$ is a convolution operator such as GRAPPA or SPIRiT. To tackle the optimization problem with two unknowns, $\mathbf{x}_l$ and $\mathbf{g}_l$, under the framework of the alternating minimization algorithm, we decompose Eq. (4) into the two simplified subproblems: 1) Kalman smoother calibration, and 2) sparse signal recovery:

$$\mathbf{P1:}\ \mathbf{g}_l = \min_{\mathbf{g}_l} \frac{1}{2} \|\tilde{\mathbf{S}}\mathbf{g}_l - \tilde{\mathbf{y}}_l\|_{\tilde{\mathbf{R}}_l^{-1}}^2 + \frac{1}{2}\sum_{t=1}^{T} \|\mathbf{g}_{l,t} - \mathbf{g}_{l,t-1}\|_{\mathbf{Q}_{l,t}^{-1}}^2 \qquad \mathbf{(5)}$$

$$\mathbf{P2:}\ \mathbf{x}_l = \min_{\mathbf{x}_l} \boldsymbol{\lambda}_1 \|\mathbf{F}_t \mathbf{F}_s \mathbf{x}_l\|_1 + \frac{\lambda_2}{2} \|\mathbf{x}_l - \mathcal{A}(\mathbf{y}, \mathbf{g}_l)\|_2^2 \qquad \mathbf{(6)}$$

where $\tilde{\mathbf{y}}_l = [\mathbf{y}_l^T\ \mathbf{x}_l^T]^T$, $\tilde{\mathbf{S}} = [\mathbf{S}^T\ \hat{\mathbf{S}}^T]^T$, $\tilde{\mathbf{R}}_l = \mathbf{diag}(\mathbf{R}_l, \boldsymbol{\lambda}_2^{-1}\mathbf{I})$, and $\mathbf{x}_l$ and $\hat{\mathbf{S}}$ are the estimated signal vector and its corresponding source matrix in k-t space at the previous step. The proposed minimization, which alternates the Kalman smoother calibration with the sparse signal recovery, continues until the objective function converges.

Methods and Results: To validate the temporally variant kernels representing time-varying coil sensitivity, real-time cardiac cine data (128x128x48) in k-t space were prospectively acquired using 8-channels for both normal and coached free breathing at the net reduction factor of 4.0. The images reconstructed by each method in the coached free breathing (Fig. 2) produces large aliasing artifacts and obscures temporal dynamics than those reconstructed in the normal free breathing (Fig. 1). Although k-t SPARKS is slightly impaired by breathing patterns, it improves image quality exhibiting less aliasing artifacts and delineating respiratory-cardiac motions compared to the existing methods (Fig. 2).

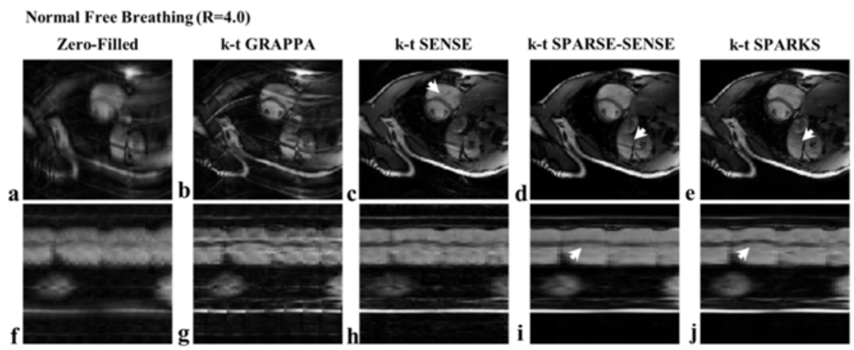

Figure 1. *Spatial and temporal dynamic images for normal free-breathing prospective cardiac cine data*

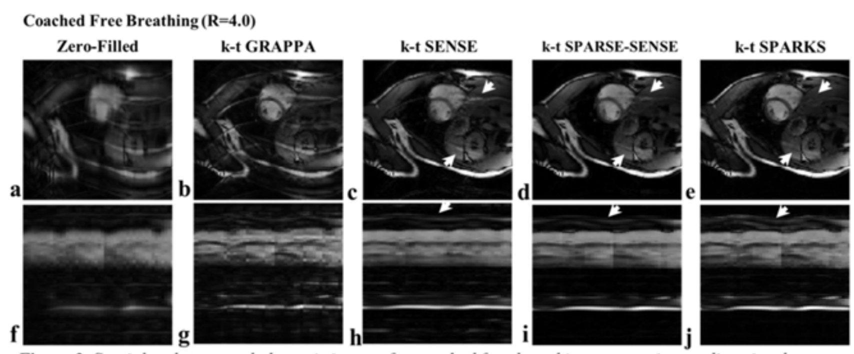

Figure 2. *Spatial and temporal dynamic images for coached free-breathing prospective cardiac cine data*

Conclusion: we successfully demonstrated the effectiveness of the proposed k-t SPARKS in suppressing aliasing artifacts and preserving temporal dynamics even with a small amount of calibration signals. It is expected that the proposed k-t SPARKS widens its clinical applications without the need for breath-holding.

References: 1. Tsao et. al., MRM, 50:1031-42, 2. Huang et. al., MRM, 54:1172-84, 3. Otazo et. al., MRM, 64:767-76, 4. Sumbul et. al. IEEE TMI, 28:1093-1104, 5. Fraser et. al., IEEE TAC, 14:387-90.