IMPULSE: A Generalized and Scalable Algorithm for Joint Design of Minimum SAR Parallel Transmit RF Pulses

Mihir Pendse[1] and Brian Rutt[1]

[1]*Radiology, Stanford University, Stanford, CA, United States*

TARGET AUDIENCE: Researchers and engineers interested in parallel transmit and RF safety

PURPOSE: To guarantee patient safety in parallel transmission (pTx), local SAR must be considered in pTx RF pulse design to ensure that the regulatory limit is not exceeded at any voxel. Since the number of voxels in the grid used to estimate SAR is large ($> 10^5$), prior optimization methods have relied on a precomputed compression step to form a much smaller ($\sim 10^3$) set of Virtual Observation Points (VOPs) [1,2] to make the problem tractable enough to allow on-scanner pTx pulse design. Here we describe the Iterative Minimization Procedure with Uncompressed Local SAR Estimate (IMPULSE) method that performs pulse optimization using the complete SAR estimate at all voxels without compression, thus resulting in savings in precomputation time and elimination of VOP overestimation error. Additionally, IMPULSE can design pTx pulses that achieve temporal averaging of local SAR hotspots, or "SAR hopping" [3], in a particularly efficient manner. Finally, we show that IMPULSE can tackle nonconvexities of the design problem efficiently, allowing for optimization of k-space trajectory as well as target phases in cases where only desired excitation magnitude is specified. Most importantly, even while considering all these aspects of the pulse design problem, the computation time with IMPULSE is *less* than alternative methods and applicable for on-scanner real-time pulse design.

† minSAR Optimization Problem

MINIMIZE: $SAR(\boldsymbol{B}) = \max_{r=1,\dots,N_R}\left(\sum_{s=1}^{N_s}\sum_{k=1}^{N_k} \boldsymbol{b}_{(s)(k)}^{\boldsymbol{H}} \boldsymbol{R}_r \boldsymbol{b}_{(s)(k)}\right)$

SUBJECT TO: $\forall s \; FAI\left(\boldsymbol{b}_{(s)(1)}, \dots, \boldsymbol{b}_{(s)(N_k)}, \mathbb{K}_{(s),(1)}, \dots, \mathbb{K}_{(s),(N_k)}, \varphi_{(s)}\right) \le \varepsilon$

‡ IMPULSE algorithm

GIVEN: $i = 0$; $\boldsymbol{Y}^{(i)}, \boldsymbol{U}^{(i)}, \boldsymbol{Z}^{(i)}$

REPEAT:

$\boldsymbol{Y}^{(i+1)} = \mathrm{argmin}_{\boldsymbol{B}}\left(SAR(\boldsymbol{B}) + \frac{\rho}{2}\|\boldsymbol{B} - \boldsymbol{Z}^{(i)} + \boldsymbol{U}^{(i)}\|_2^2\right)$ ⊞

$\boldsymbol{Z}^{(i+1)} = \mathrm{proj}_{FAI}\left(\boldsymbol{Y}^{(i+1)} + \boldsymbol{U}^{(i)}\right)$ ⊠

$\boldsymbol{U}^{(i+1)} = \boldsymbol{U}^{(i)} + \boldsymbol{Y}^{(i+1)} - \boldsymbol{Z}^{(i+1)}$

UNTIL: $\boldsymbol{Y}^{(i+1)} = \boldsymbol{Z}^{(i+1)}$

THEORY: We wish to determine the complex amplitudes to apply to each of N_c transmit channels during N_s different pTx pulses, each with N_k subpulses corresponding to multiple k-space locations. While IMPULSE can be generalized to arbitrary trajectories and excitation patterns, we consider the specific case of a small-tip slice-selective excitation with spokes trajectory. The channel weights can be denoted by the matrix $\boldsymbol{B} \in \mathbb{C}^{N_c x N_k N_s}$ composed of columns $\boldsymbol{b}_{(s)(k)} \in \mathbb{C}^{N_c}$ that represent the complex amplitudes for the kth subpulse of the sth pulse. In addition to $\boldsymbol{B}$**,** the flip angle inhomogeneity (FAI) is a function of spokes locations ($\mathbb{K}$) and phase of the target pattern (φ). We must find $\boldsymbol{B}$, $\mathbb{K}$, and φ that produce an FAI that is less than some user-specified threshold (ε) for each slice. Subject to this constraint, the $\boldsymbol{B}$ that produces minimum SAR penalty is desired. This "minSAR" problem can be formulated with a convex cost function SAR and N_s nonconvex constraints $FAI_{(s)}$ (as seen in † above). SAR is a time-averaged function expressed as the maximum of N_R quadratic forms where each matrix $\boldsymbol{R}$ corresponds to either the local SAR matrix at each voxel, global SAR matrix, or average power (identity) matrix at each channel. Each $\boldsymbol{R}$ matrix is normalized by both the duration of each subpulse and the regulatory limit for the corresponding hardware or safety metric so that $SAR \le 1$ indicates that all SAR constraints are satisfied. After obtaining the minimum SAR pulse from optimization, the RF duty cycle can be adjusted to ensure that $SAR \le 1$. One approach to solve this nonconvex problem is through sequential quadratic programming (SQP) whereby a generic convex optimization solver is used to iteratively solve the relaxed minimum SAR problem with fixed φ and $\mathbb{K}$ followed by a separate nonconvex update of φ and $\mathbb{K}$ with fixed $\boldsymbol{B}$. Because of the poor scalability with N_R of generic convex optimization solvers, VOPs or some other compression or acceleration technique must be used to reduce N_R or computation time with SQP. We introduce here the IMPULSE algorithm (see ‡ above) that solves the minimum SAR problem using the theory of alternating direction method of multipliers (ADMM) [4] to decompose the optimization problem into a SAR update (see ⊞) and an FAI update (see ⊠) which are done sequentially at each iteration until a convergence condition is satisfied. The SAR update consists of an unconstrained minimization of a convex piecewise quadratic function consisting of the SAR cost function plus a regularization term. This step can be performed efficiently through the use of bundle methods based on construction of a piecewise linear approximation of $SAR(\boldsymbol{B})$ [5,8]. The FAI update consists of a projection [6] of a point onto the nonconvex feasible set of pulses that satisfy both the FAI and instantaneous power constraints. Since these constraints are independent for each s, this step can be parallelized so that the duration of each iteration is independent of N_s.

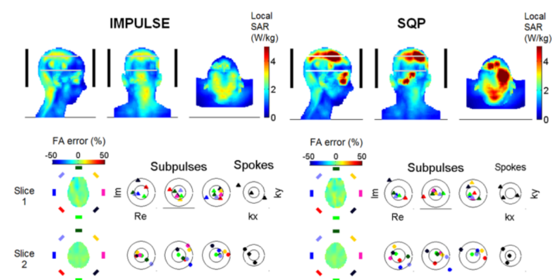

Figure 1: Design of two 3-spoke pulses with both IMPULSE and SQP.

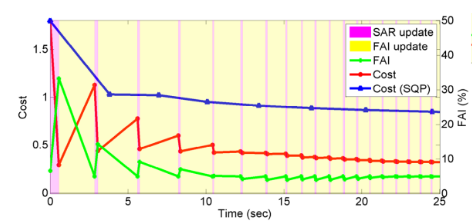

Figure 2: Convergence of the IMPUSLE algorithm and comparison with SQP.

METHODS: The minSAR problem was solved with IMPULSE and with SQP using VOPs. Electric fields were simulated on the Ella body model (SEMCAD X, Virtual Family) with an 8 channel loop array. A 3-spoke small-tip pulse was optimized at each of two slices with a FAI tolerance of $\varepsilon = 5\%$. Additional experiments were done with IMPULSE using 100 slices as well as 4 body models for a robust multi-body-model SAR estimate [7] to demonstrate negligible impact on computation time with large values of N_s and N_R. In all cases, initial spokes selection was performed using the SAR-unaware interleaved greedy and local algorithm [9] but spoke locations were further updated with each SAR-aware algorithm.

RESULTS: As indicated in Figure 1, IMPULSE results in less than half the peak local SAR compared to SQP/VOPs without sacrificing excitation homogeneity. Figure 2 demonstrates that convergence occurs in under 25 seconds of computation, faster than the SQP/VOPs method. Figure 3 indicates that IMPULSE can be performed in approximately constant time even with large number of slices or number of SAR voxels.

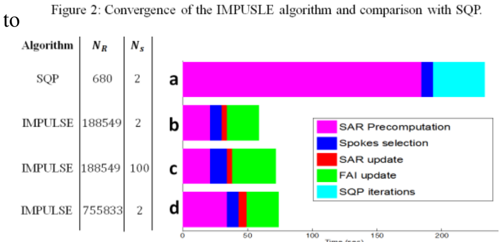

Figure 3: Computation time from obtaining electric fields to final optimized pTx pulse for four different cases each with 5% FAI tolerance: **(a)** SQP with 680 VOPs, **(b)** IMPULSE without VOP compression, **(c)** uncompressed IMPULSE with 100 slices, **(d)** IMPULSE on two slices with four body models used for more robust SAR estimate.

DISCUSSION: Three main benefits of the IMPULSE algorithm are clear:

1. Ability to optimize SAR directly without compression. Because of the efficient bundle algorithm for the SAR update, arbitrarily many terms in the objective function are possible, allowing for reduction in both offline and online computation time as well as elimination of VOP overestimation error and greater ultimate SAR reduction.

2. Better (nonconvex) optimization of spokes locations and target phases. The nonconvex update is integrated into the FAI update as a projection operation rather than a completely separate minimization problem with SQP. Since projection operations (even nonconvex ones) can be performed relatively reliably, IMPULSE treats nonconvexities more rigorously.

3. Temporal averaging of SAR hotspots through parallelization across slices. Since the optimization is done jointly for all pulses/slices in the sequence, temporal averaging of SAR hotspots is guaranteed. Importantly, this can be accomplished with little or no increase in computation time.

CONCLUSION: IMPULSE is an efficient algorithm that minimzes local SAR, optimizes k-space trajectory, and allows for temporal hotspot averaging in pTx pulse design. The described procedure can be generalized to more complex excitation schemes by only modifying the FAI projection operation

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