A Vectorized Formalism for Efficient SAR Computation in Parallel Transmission

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TARGET AUDIENCE: Researchers and engineers interested in RF safety and parallel transmit

PURPOSE: In high field parallel transmit (pTx) pulse design, efficient methods for computing the feasibility of a given pulse with respect to time-averaged constraints including local SAR, global SAR, and average transmitted power are necessary for SAR-aware pulse optimization. Among these quantities, computation of local SAR is the most demanding as it requires considering all ($>10^5$) voxels in the electric field grid. The Virtual Observation Points (VOP) compression method [1] was proposed as one approach for making this computation efficient, but has drawbacks of requiring significant precomputation and introducing an overestimation error. We introduce a new formalism involving a single normalized SAR cost function incorporating all time-averaged constraints and then describe a vectorized oracle that can efficiently evaluate this function and its derivatives in a minimum number of operations. In contrast with VOPs, precomputation time for constructing the oracle is minimal and the function is evaluated exactly without overestimation. The oracle can be integrated into novel pTx algorithms that rely on bundle methods for constructing piecewise linear surrogates for the cost function [2] as well as new areas of research such as design of hyperthermia pulses [3].

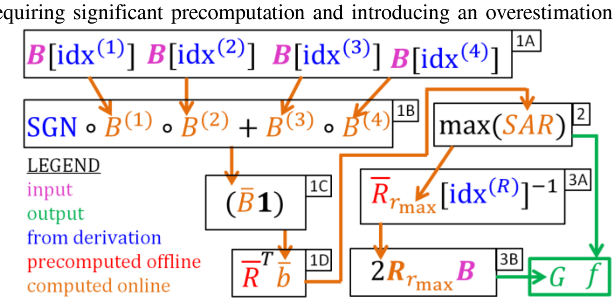
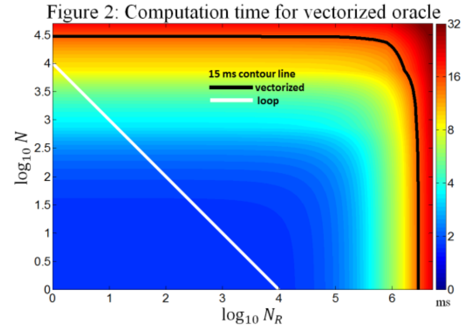


Figure 1: Procedure for vectorized oracle computation

THEORY: We consider a pulse sequence of duration T whose complete RF waveform is parameterized by $N_c N$ complex variables corresponding to the complex amplitudes applied to each of N_c transmit channels at each of N subpulses at different excitation k-space locations throughout the scan. These variables can be characterized by a matrix $\boldsymbol{B} \in \mathbb{C}^{N_c \mathrm{x} N}$ whose nth column denoted $\boldsymbol{b}_n$ represents the channel amplitude at the nth subpulse. It has been shown [4] that local SAR, global SAR, and average power for a given channel weighting can all be expressed as positive semidefinite (PSD) quadratic forms $\boldsymbol{b}_n^H \boldsymbol{R}_r \boldsymbol{b}_n$ where $\boldsymbol{R}_r$ is either a local SAR matrix at one of N_v voxels, a global SAR matrix for the entire exposed mass, or an average power matrix at one of N_c channels. If each $\boldsymbol{R}_r$ includes a normalization by the regulatory limit for the corresponding quantity as well as the duration of the scan (T), the expression $f(\boldsymbol{B}) = \max_{r=1,\dots N_R}(SAR)$, where the vector SAR is defined as $SAR_r = \sum_{n=1}^{N} \boldsymbol{b}_n^H \boldsymbol{R}_r \boldsymbol{b}_n$, serves as a time-averaged cost function for all $N_R = N_v + N_c + 1 \approx N_v$ safety/hardware constraints. Because of the normalization, when $f > 1$, some regulatory limit is violated. Pulse design problems can be formulated [2] to minimize f subject to some constraint on excitation accuracy. The optimal value of f then represents the amount by which the RF duty cycle can be scaled to abide by regulatory limits. To perform such an optimization it is necessary to repeatedly evaluate not only f but also its gradient (or more correctly its subgradient since f is nondifferentiable). We denote the subgradient for a particular $\boldsymbol{b}_n$ by the vector $\boldsymbol{g}_n \in \mathbb{C}^{N_c}$ and define the matrix $\boldsymbol{G}$ as $[\boldsymbol{g}_1 \ \cdots \ \boldsymbol{g}_N]$. We desire an efficient oracle that receives $\boldsymbol{B}$ and outputs f and $\boldsymbol{G}$ in the minimum amount of time. The general procedure (Figure 1) to do this is (1) compute SAR from $\boldsymbol{B}$, (2) compute f and $r_{\max}$ from SAR, where $f = \max(SAR) = SAR_{r_{\max}}$, (3) compute a subgradient through analytical differentiation of the cost function at the peak voxel: $\boldsymbol{G} = 2\boldsymbol{R}_{r_{\max}}\boldsymbol{B}$. Among these steps, (1) is the most costly and extremely slow if done inefficiently in a loop over r and n. This step can be made vastly more efficient by exploiting the symmetry of $\boldsymbol{R}_r$ to evaluate the quadratic form $\boldsymbol{b}_n^H \boldsymbol{R}_r \boldsymbol{b}_n$ in a vectorized manner. It can be shown that the entire vector SAR can be obtained through a single matrix vector multiplication $\bar{R}^T \bar{b}$ as in (1 A-D) in Figure 1. The first step (1A) consists of creating new *real*-valued matrices $B^{(\cdot)} \in \mathbb{R}^{N_c^2 \mathrm{x} N}$ whose nth column consists of the real and imaginary components of $\boldsymbol{b}_n$ at indices $\mathrm{idx}^{(\cdot)}$ (whose identity can be derived). Next, $\bar{b}$ is found by first forming $\bar{B}$ through elementwise operations on a matrix SGN (whose identity can be derived) and $B^{(\cdot)}$ (1B) then time-averaging the columns of $\bar{B}$ (1C). SAR is found through a dense matrix vector multiplication $\bar{R}^T \bar{b}$ (1D) where the rth column of $\bar{R} \in \mathbb{R}^{N_c^2 \mathrm{x} N_R}$ is found by selection of the N_c^2 unique components of the PSD SAR matrix $\boldsymbol{R}_r$ at indices $\mathrm{idx}^{(R)}$. In fact, $\bar{R}$ can be precomputed directly from electric fields and tissue properties, entirely bypassing construction of SAR matrices. The single SAR matrix $\boldsymbol{R}_{r_{\max}}$, which is the Hessian of f, can be reconstructed from $\bar{R}_{r_{\max}}$, the $r_{\max}$th column of $\bar{R}$ (3A) and used in subgradient computation (3B).

METHODS: We report times for computing f and $\boldsymbol{G}$ over range of values for N and N_R. GPU acceleration was implemented by precomputing $\bar{R}$ and storing it on the GPU (NVIDIA GeForce GTX 670). For each pulse matrix, $\bar{b}$ was constructed from $\boldsymbol{B}$ and then transferred to the GPU for dense matrix multiplication. Accuracy of the vectorized method was established by verifying that an identical SAR vector resulted when using a loop. Since we would like the oracle computation time to be about 15 ms for tractability of optimization, the 15ms contour of the (N, N_R) space was computed and compared to use of a loop.

Figure 2: Computation time for vectorized oracle


RESULTS: As seen in Figure 2, computation time scales with $N_R N$ when using a loop; therefore, even with just one subpulse ($N = 1$), the maximum possible value of N_R is 10^4 which is an order of magnitude less than the number of voxels in a body model. Using the vectorized method, computation time is approximately constant regardless of N or N_R. Oracle computation time is within an acceptable range for $N \leq 10^4$ and $N_R \leq 10^6$.

DISCUSSION: The vectorized formulation is able to evaluate the normalized SAR cost function and its derivatives in approximately constant time regardless of number of subpulses in the sequence or number of voxels in the local SAR estimate. The lack of dependence on N arises from the fact that time-averaging (formation of $\bar{b}$) is performed *before* dense matrix multiplication. The lack of dependence on N_R results because $\bar{R}^T \bar{b}$ is evaluated on the GPU which parallelizes the computation across the N_R columns of $\bar{R}$. Construction of $\bar{R}$ from patient-specific multichannel electric field maps requires 20 seconds, approximately the same time as needed for construction of spatially averaged SAR matrices and much shorter than VOP compression time (several minutes). Since a typical body model has $\sim 10^5$ voxels, Figure 2 indicates that using the vectorized oracle, optimization is possible without a compression step. Furthermore, it may be possible to increase robustness in the SAR estimate by including *multiple* body models [5] to reduce sensitivity to patient/body model mismatch. A maximum N of 10^4 shows that it is possible to design very demanding RF pulse sequences with a single time-averaged SAR estimate (eg, $N = 10^4$ could correspond to a trajectory with 10 kT-points in each of 1000 separate excitations).

CONCLUSION: We present a new SAR formalism for handling the large computational demands of incorporating patient-specific SAR constraints into pulse optimization that has significant benefits to prior methods such as VOPs in terms of both efficiency and accuracy.

References: [1] Eichfelder et. al. MRM 2011;66:1468–1476. [2] Pendse, Abstract ID 4996, ISMRM 2015 [3] Pendse, Abstract ID 4357, ISMRM 2015 [4] Zhu. et al. MRM 2012; 67:1367–1378. [5] Pendse, Abstract ID 5008, ISMRM 2015 **Acknowledgement**: Research support from NIH P41 EB015891, NIH 1 S10 RR026351-01Al, GE Healthcare.