Hierarchically Semiseparable Generalized Encoding Matrix Compression for Fast Distortion Corrected Inverse Imaging

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TARGET AUDIENCE: Neuroimaging scientists and clinicians interested in efficient and accurate model-based reconstruction of non-Cartesian data.

PURPOSE: Reconstruction of images from non-Cartesian data can be a computationally demanding problem. Iterative numerical solutions often involve repeated evaluation of Discrete Fourier or NUFT [1] operators, sensitivity profiles, and other physical MR parameters [2-6]. In this work, we introduce a direct method for computing an approximate inverse of a generalized encoding matrix to provide fast and accurate reconstruction. This compact inverse model can be used to quickly map acquired data directly to an unaliased and off-resonance corrected image. This is accomplished through Hierarchically Semiseparable (HSS) modeling of the encoding matrix [7,8]. HSS modeling can be computed prior to data collection and is ideal for time series reconstruction, e.g. fMRI, cardiac imaging, and MR fingerprinting. We demonstrate the efficiency of our approach for the reconstruction of spiral data incorporating B0 distortion correction and compare its performance to standard gridding approaches.

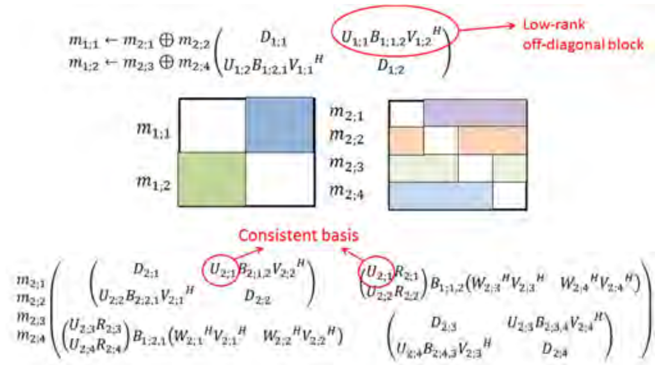

Fig. 1. Hierarchically Semiseparable model of image encoding matrix.

METHOD: FFT based algorithms for image reconstruction of Cartesian data have been successfully incorporated into clinical settings. However, model-based reconstructions of non-Cartesian trajectories that include non-linear effects can be computationally prohibitive. A typical parallel imaging (PI) forward model for model-based image reconstruction [3,9] is as follows: $s_l(t) = \int c_l(\vec{r}) e^{-i\Delta\omega_0(\vec{r})t} e^{-2\pi i\vec{k}(t)\cdot\vec{r}} f(\vec{r}) d\vec{r}$ (Eq. 1). The signal at channel l and time t is a function of the receive coil profiles $c_l(\vec{r})$, the off-resonance phase accrual $\Delta\omega_0(\vec{r})t$, and the Fourier encoding $2\pi i\vec{k}(t)\cdot\vec{r}$. Iterative solvers [2-6] are typically used to solve the forward model (1) to estimate the image $f(\vec{r})$. Many of these methods can be computationally demanding due to repeated discrete Fourier operator evaluation. More efficient NUFT [1] operators have been used as an approximation. However, while utilizing NUFT in (1) the B0 inhomogeneity effect will need to be approximated using time-interval basis functions [2]. Alternatively, HSS modeling [7,8] can be applied to (1) in order to compress voxel-voxel interactions while preserving the accuracy of the encoding. Fig. 1 shows two levels of the HSS model reduction scheme applied to the encoding matrix. The diagonal blocks $\{D\}$ represent smaller encoding matrices, for subsets of voxels, that are coupled through low-rank basis terms $\{U, R, B, V, W\}$. The low rank basis coefficients are determined through a bottom-up traversal of the voxel subgroups. For example, SVD or rank revealing QR would first be used to determine the low-rank basis coefficients for the encoding matrix in regions $m_{2;1}$ and $m_{2;2}$, which will then be used to describe the encoding matrix for the region $m_{1;1}$. After HSS modeling, recursion can then be used to form an implicit representation [7] for the inverse encoding matrix. It is important to note that the HSS encoding model can be constructed based solely upon the gradient trajectory and pre-scan information.

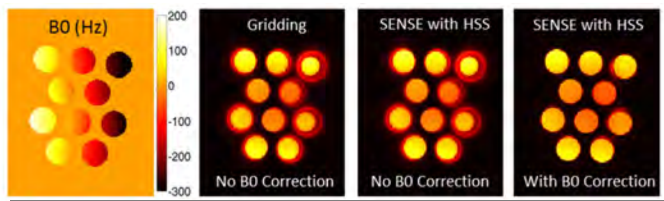

Fig. 2. Reconstruction of fully sampled 48-shot spiral phantom data. Standard gridding and SENSE reconstructions are compared. Reduction of blurring artifacts with distortion corrected SENSE (using HSS) is shown.

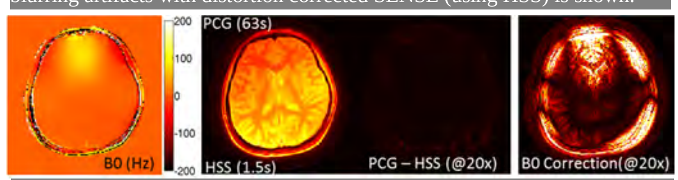

Fig. 3. Reconstruction of 48-shot spiral in vivo data. The time/accuracy of SENSE using HSS and PCG reconstructions (incorporating B0 estimate) are compared. The distortion corrected regions of the image are shown.

RESULTS: The performance of the HSS scheme was demonstrated using spiral data acquired both in a phantom and in vivo from a healthy volunteer on a 3 T Siemens Skyra scanner with 16ch head array coil. 48-shot fully sampled variable density spiral data were acquired at $1.2\times1.2\times5mm^3$ resolution with 300mm FOV, and TR=11.5ms. The B0 maps were created using exponential phase fitting across multi-echo GRE data with TE=2.32ms, where the 6 echoes were separated by 0.38ms. Fig. 2. shows the standard NUFT based gridding [1] and SENSE [9] reconstructions without correcting for the B0 field applied across the cylindrical phantoms. The large blurring artifacts are significantly reduced with the inclusion of B0 map into the forward model. Fig. 3 shows the speed and accuracy of the HSS model, which provides nearly identical reconstruction results 40× faster than Jacobi Pre-conditioned Conjugate Gradient (PCG) [5], both tuned for 10^{-4} tolerance with B0 inhomogeneity built in. Given the large number of multi-channel spiral observations (>1.7 million), PCG on the pre-computed dense encoding matrix gave superior performance to repeated NUFT evaluations. The deblurring effect in areas of large B0 inhomogeneity is shown in the difference image on the right of Fig. 3.

DISCUSSION and CONCLUSION: HSS modeling of the encoding process can effectively be used to directly reconstruct unaliased and deblurred non-Cartesian data. This bottom-up compression strategy should allow for the modeling of large encoding matrices (e.g. 3D or high resolution 2D), as only small portions of the encoding matrix need to be generated and stored in memory at any stage of the process. Although there is overhead in computing the HSS model of the encoding matrix (<40min in serial MATLAB for the examples in this work), the bottom-up strategy lends itself very well to parallelization as many of the compression operations can be computed concurrently. When considering time-series reconstructions such as fMRI, cardiac imaging, and MR fingerprinting this fixed overhead will be very small in comparison to the reconstruction time for the images, see Fig. 3. With the integration of parallel computing, HSS modeling enables clinically relevant reconstruction across many imaging applications.

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