JOINT COMPRESSED SENSING AND SPARSE PHASE RETRIEVAL: RECONSTRUCTION FROM A COMBINATION OF COMPLEX AND MAGNITUDE-ONLY K-SPACE MEASUREMENTS

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TARGET AUDIENCE: Scientists working on accelerated imaging, image reconstruction, compressed sensing and sparse models.

INTRODUCTION: Compressed sensing (CS), which utilizes the sparsity of images for reconstruction [1], has been used in a number of applications in MRI. Sparse phase retrieval tackles a similar problem, where a sparse signal is reconstructed from only the magnitude of the measurements [2]. Recently, it has been shown that if one has access to a combination of linear measurements (as in CS), and magnitude-only measurements, then two random magnitude measurements hold the same information as one random complex linear measurement [3].

This suggests that additional magnitude k-space information (which cannot be used in a conventional CS reconstruction due to the non-linearity of these measurements) may be used to improve accelerated reconstruction. Such additional information may be available in various scenarios at no additional scan time: Translational motion preserves k-space magnitude, and motions such as the respiratory motion of the heart are well-approximated as translational [4]; using approximate Hermitian symmetry [5], the k-space point at (k_x,k_y) has similar magnitude to that of $(-k_x,-k_y)$.

In this study, we sought to introduce a new sparsity-regularized reconstruction paradigm based on using conventional complex k-space measurements, along with magnitude-only k-space measurements available as side information.

METHODS: THEORY AND IMAGE RECONSTRUCTION: Suppose we have complex k-space data $\mathbf{y} = \mathbf{E}_\Omega\mathbf{m}$, along with magnitude-only k-space side information $\mathbf{r} = |\mathbf{E}_\Lambda\mathbf{m}|$, where $\mathbf{E}_\Omega$ and $\mathbf{E}_\Lambda$ are system matrices sampling the k-space points specified by Ω and Λ respectively, and $|\cdot|$ is the element-wise absolute value. We reconstruct a sparsity-regularized $\mathbf{m}$ from $\mathbf{y}$ and $\mathbf{r}$ by solving the objective function:

$$\arg\min_{\mathbf{m}} \left\| \mathbf{y} - \mathbf{E}_\Omega \mathbf{m} \right\|_2^2 + \lambda_{mag} \left\| \mathbf{r} - \left| \mathbf{E}_\Lambda \mathbf{m} \right| \right\|_2^2 + \lambda_{sparse} \Phi(\mathbf{m}).$$

This is solved using an augmented Lagrangian technique using an alternating direction method approach, similar to the one in [6]. For comparison, images were also reconstructed with CS reconstruction that only uses complex k-space data. For radial data, NUFFT [7] is used in implementing $\mathbf{E}_\Omega$ and $\mathbf{E}_\Lambda$. Total variation (TV) regularization is utilized as $\Phi(\mathbf{m})$ for all reconstructions using the same parameters.

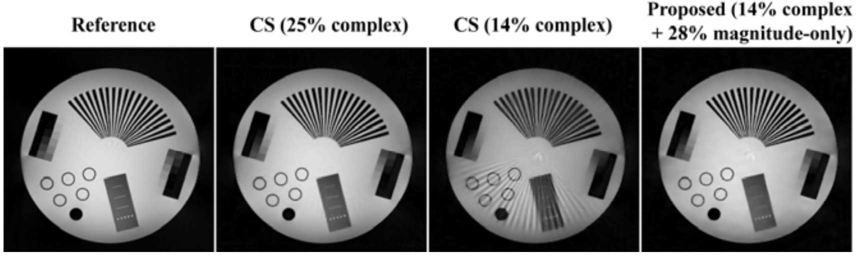

Figure 1: *Phantom reconstructions with radial imaging. CS with 25% of spokes is comparable to the reference, but CS with 14% of spokes exhibits substantial streaking artifacts. With the use of 28% additional magnitude-only k-space data, these artifacts are removed.*

PHANTOM IMAGING: To test the feasibility, phantom imaging was performed using a body coil (NSA=10 for sufficient SNR) Radial Imaging: A 2D GRE acquisition was performed with resolution=1×1 mm^2, FOV=300×300 mm^2. The data was retrospectively (uniform angularly) undersampled to 25% and 14% of the Nyquist rate for CS reconstruction. For the proposed technique, 14% complex data and 28% additional magnitude-data (from the same dataset) were utilized. Cartesian Imaging: 2D spin echo data was acquired with resolution=1×1 mm^2, FOV=270×270 mm^2. 3-fold retrospective undersampling was used (patterns shown in **Fig. 2**). For the proposed technique, the additional magnitude information was estimated as $\mathbf{r}(-k_x,-k_y) \approx |\mathbf{y}(k_x,k_y)|$.

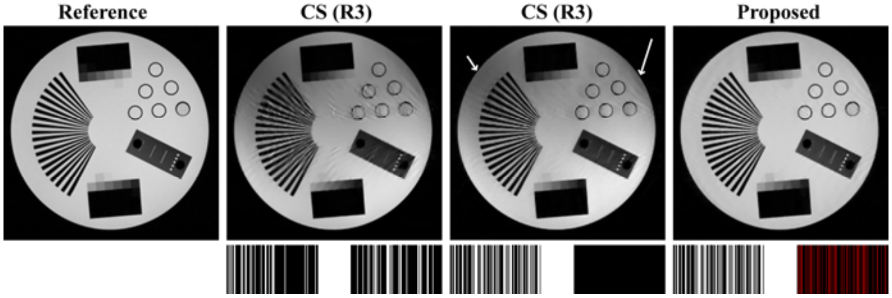

Figure 2: *Phantom reconstructions with Cartesian imaging, as well as the associated undersampling patterns depicted below (cropped in k_x direction, white = complex, red = magnitude-only). For the proposed method, $\mathbf{r}(-k_x,-k_y)$ is estimated as $|\mathbf{y}(k_x,k_y)|$. Additional magnitude k-space data removes aliasing artifacts and improves signal homogeneity (arrows), without any increase in the scan time.*

IN-VIVO IMAGING: In-vivo feasibility was tested using a 2D SSFP cine acquisition with resolution= 1.7×1.7 mm^2, FOV=300×300 mm^2, 20 cardiac phases, with a 32-channel cardiac coil. The acquisition had 176 spokes, which was retrospectively undersampled to 16 spokes for CS reconstruction of a systolic phase image. Additional magnitude-only k-space data were utilized from 10 other phases (5 diastolic phases, 160 magnitude-only spokes) for the proposed method.

RESULTS: **Fig. 1** depicts the phantom reconstructions from radial imaging. Additional magnitude-only data improves the reconstruction by removing streaking artifacts. We note that the undersampling rates and improvement are in good agreement with the predicted theoretical trade-off. **Fig. 2** shows the phantom reconstructions from Cartesian imaging. The additional magnitude-only k-space data improves the reconstruction at no additional scan time. **Fig. 3** depicts the results from in-vivo imaging. CS reconstruction with 16 spokes leads to streaking artifacts and blurring. These artifacts are largely removed by including additional k-space magnitude data from other cardiac phases.

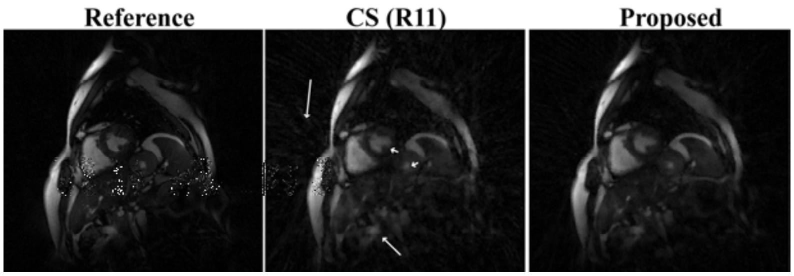

Figure 3: *In-vivo reconstructions with radial imaging. CS image, reconstructed from 16 radial spokes of a systolic phase image, exhibits blurring and streaking artifacts. These artifacts are largely removed using the proposed reconstruction with additional k-space magnitude information from other cardiac phases, and the myocardium appears sharper.*

CONCLUSIONS: We have demonstrated the feasibility of a sparse image reconstruction framework from a combination of complex and magnitude-only k-space measurements, which may outperform conventional CS reconstruction. We have also discussed example applications where such magnitude-only measurements may be available at no additional scan time, including magnitude k-space data from other cardiac phases or from partial Fourier data.

REFERENCES: [1] Lustig, MRM, 2007; [2] Akçakaya, IEEE TSP, in press; [3] Akçakaya, IEEE SPL, in press; [4] Scott, Radiology, 2009; [5] McGibney, MRM, 1993; [6] Akçakaya, PLoS ONE, 2014; [7] Fessler, IEEE TSP, 2003. **ACKNOWLEDGEMENTS:** Grant support from NIH K99HL111410-01.