Use of Adaptive Diffusion Filters to Estimate In-Vivo Conductivity Images from $B_1^+$ Maps
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Target audience: This work is relevant to those interested in electric property tomography.

Introduction: Non-invasive Electric Properties Tomography (EPT) is believed to play a great role in local SAR estimation, malicious tissue identification and electric cancer therapy. Recent EPT methods are based on variations of the Maxwell’s equations in which the electric property maps are computed from the transmit RF field ($B_1^+$) maps assuming homogeneous media distribution [1]. The relation between $B_1^+$ and $\sigma$ is governed by the Helmholtz equation shown in Eq. (1). Numerical solutions to this equation in previous studies [2-3] has shown favorable results in noise-free cases. They used modified versions of the discrete Laplacian operator ($\nabla^2$) to deal with noisy $B_1^+$ maps, but this reduces the precision of the Laplacian estimation degrading the accuracy of the electrical property maps. Instead, we propose to use a non-linear anisotropic diffusion filter (ADF) to denoise the $B_1^+$ maps prior to the Laplacian computation.

Methods: We used the iterative 3D ADF shown in Eq. (2) to obtain a denoised version of the $B_1^+$ maps ($\tilde{B}_1^+$) where $\Delta$ is the factor of diffusion speed, $k$ is the noise identification factor for a given pixel, $d$ is the direction of the gradient $\nabla_d$ [4]. After denoising, we used the conventional central difference equations along the x-, y- and z-directions to compute the second order derivatives, and then added them to get the Laplacian. An example for the x-directional derivative is shown in Eq. (3) where $h_x$ represents the x-directional pixel size. After Laplacian computations, we used Eq. (1) to get the conductivity maps. We also implemented the numerical solutions proposed by van Lier [2] and Bulumulla [3] for performance comparison. Van Lier’s method is based on the noise-robust Laplacian 3-D kernel operators (the size of 5x5x3 for the lateral direction and 7x3x3 for the in-plane directions) while Bulumulla’s method is based on skip factors in computing the second order derivatives. We performed FDTD simulations on a high-pass birdcage coil with 16 rungs tuned at 123.5 MHz using SEMCAD X (SPEAG, Switzerland). We set an isotropic grid for a FOV of 300x300x300mm with a total matrix size of 128x128x32. We computed $B_1^+$ field inside a double cylindrical phantom (radius=47mm, height=158mm) that had electric conductivities of 0.2 and 1.8 S/m (L and H in Fig. 1a, respectively) and a relative permittivity of 76.7. We also computed $B_1^+$ field in the synthetic human head model with the same pixel size. We performed in-vivo imaging experiments at 2.9T MRI using the double angle method (DAM) to obtain $B_1^+$ magnitude and phase maps of a volunteer’s head placed in a high-pass birdcage coil that had the same shape as the one in the FDTD simulation (scan parameters: FOV=300mm, THK=10mm, matrix size=128x128, TE=20ms, TR=5s, FA=60°/120°).

Results: From the FDTD simulation data, we extracted the magnitude and phase maps of the $B_1^+$ field. We computed the conductivity maps by $\sigma = J/E$ as shown in Fig. 1a. Figures 1b-1d show the conductivity maps computed by the three methods. The top images have been computed from the noise-free $B_1^+$ maps while the bottom images from the $B_1^+$ maps after adding white Gaussian noise (SD=0.005 when peak $B_1^+$ is normalized to one). Figure 2 shows the conductivity maps of a human head model (Fig. 2b-2d) along with the reference map (Fig. 2a) computed from the noisy $B_1^+$ maps (SD=0.005). As can be seen from the figures, all the three methods give good estimation from the noise-free $B_1^+$ maps, but the proposed method works better in noisy cases. Figure 3 shows the conductivity maps (Fig. 3b-3d) in the volunteer’s head along with the corresponding T1-weighted image (Fig. 3a). We observed that 3 iterations were enough for denoising the simulated $B_1^+$ maps, while 8 iterations were necessary for denoising the measured $B_1^+$ maps. The mean conductivity values along with the standard deviations at different homogeneous regions of 200 pixels are shown in Table 1.

Discussion and conclusions: The adaptive diffusion filter effectively removed the noise from the $B_1^+$ maps while preserving important detail structures allowing us to use a more conventional central difference Laplacian operator that worked on three consecutive neighboring pixels. This enabled us to obtain higher resolution conductivity maps for in-vivo EPT applications. From both simulation and experiments, we observed higher similarity between the reference conductivity maps and the conductivity maps computed by the proposed method. High accuracy Laplacian computation with less noise effects seems to significantly improve EPT results.