Target Audience: This work should be of interest to developers and users of advanced MR image reconstruction algorithms such as compressed sensing who wish to quantitatively evaluate the performance of these algorithms with image quality metrics that reflect clinical utility.

Purpose: Traditional Cartesian MR data reconstruction has well-understood noise properties. K-space data contaminated by Gaussian electronic noise are reconstructed via the linear FFT algorithm to obtain magnitude images containing Rician noise, often well approximated as Gaussian. Modern dynamic and quantitative imaging however benefits from faster imaging, achieved by acquiring less data and compensating through parallel imaging or constrained iterative reconstructions such as compressed sensing (CS). CS algorithms are non-linear, invalidating the original linear systems-based analysis of noise in MR images and may produce images of high apparent quality measured by metrics such as pixel SNR (SNRₚ) through de-noising or other non-linear effects, while incongruently suppressing or modifying clinically-relevant image features. The purpose of this work is to develop an image quality metric based on task-specific Hotelling SNR (SNRₕ) that reflects clinical utility, overcoming limitations of SNRp and related metrics to allow for more meaningful quantitative comparisons and optimization of accelerated acquisitions and CS reconstructions.

Methods: Consider an imaged object (human or phantom) represented by a spatial distribution of MR parameters, f(r) = \{T₁(r), T₂(r), ρ(r) \...\}, and a detection task, i.e., determine the presence (hypothesis H₁) or absence (hypothesis H₀) of a clinically important object feature. The object f passes through the MR imaging system \( \mathcal{H} \) to generate k-space data k corrupted by Gaussian noise n: k = \( \mathcal{H}f(r) + n \). K-space data k is then passed to a CS reconstruction algorithm \( \mathcal{R} \) to generate image g, thus g = \( \mathcal{R}(k) = \mathcal{R}(\mathcal{H}f(r) + n) \). The Hotelling observer is the linear observer function that maximizes separability of object classes by de-correlating the noise in the image g and subsequently multiplying by a template, the expected difference in the image between hypotheses H₀ and H₁. The corresponding Hotelling observer SNR is defined by the equation

\[
\text{SNR}_h^2 = \left[ \langle g | H_1 \rangle - \langle g | H_0 \rangle \right]^2 K_g \left[ \left( \langle g | H_1 \rangle - \langle g | H_0 \rangle \right) \right]^{-1}
\]

where \( \langle g | H_j \rangle \) represents the expectation of the image under hypotheses H_j and \( K_g \) is the image noise covariance matrix. The feature of interest is assumed to be small enough to not affect the noise correlation structure. As a measure of image quality we propose the Hotelling efficiency \( \epsilon_h \), defined as the ratio of \( \text{SNR}_h \) for a fully-sampled Cartesian acquisition and \( \text{SNR}_h \) for an under-sampled CS acquisition, thus representing the loss in task performance by resorting to an under-sampled acquisition,

\[
\epsilon_h = \frac{\text{SNR}_h(\mathcal{H} = \text{under-sampled}, \mathcal{R} = \text{CS reconstruction})}{\text{SNR}_h(\mathcal{H} = \text{Cartesian fully-sampled}, \mathcal{R} = \text{FFT reconstruction})}
\]

To test this metric a simulated imaging system and high-resolution anthropomorphic digital brain phantom were utilized to generate image recon for a 2D Cartesian spin-echo sequence (TE=30ms, TR=1500ms, matrix=256x256) and under-sampled 2D pseudo-radial spin-echo sequence with the same timings (64 or 128 views). The CS reconstruction was an implementation of the Split-Bregman technique with total variation and wavelet smoothing or de-noising depending on the choice of regularization parameters. At some level de-noising will begin to negatively impact clinical method better reflects clinical utility and is not susceptible to the problems encountered when using SNRp for non-linear reconstructions.

Conclusion: We have developed and tested an image quality metric that is able to compare and optimize CS regularized reconstructions. This method better reflects clinical utility and is not susceptible to the problems encountered when using SNRp for non-linear reconstructions.


Fig 1. Calculated efficiency \( \epsilon_h \) for locations throughout the brain for 64 radial views (l) and 128 radial views (r).

Fig 2. The \( \epsilon_h \) (l) and SNRp (r) metrics for 64-view recon with high regularization vs. 128-view recon with lower regularization for each ROI shown in Fig. 1. SNRp indicates better performance with 64-view data for 74.8% of ROIs while \( \epsilon_h \) indicates 128-view data are superior for all ROIs.