Characterizing the Inherent and Noise-Induced Errors in Actual Flip Angle Imaging

M Louis Lauzon1,2 and Richard Frayne1,2

1Radiology, Hotchkiss Brain Institute, University of Calgary, Calgary, AB, Canada, 2Seaman Family MR Research Centre, Calgary, AB, Canada

Target Audience: Researchers interested in measuring and characterizing the flip angle term in an image.

Purpose: $T_1$ mapping requires knowing the flip angle, $\theta$, or equivalently $\cos \theta$; it can be determined using the TR-interleaved spoiled gradient-recalled echo AFI (actual flip angle imaging) sequence. AFI makes an approximation and uses non-linear processing, two issues that can affect both the accuracy and precision of the calculation. Our objective is to theoretically determine these errors for $\cos \theta$ (i.e., the estimate of $\cos \theta$) as a function of $T_1$ and the various AFI acquisition parameters in order to provide an overall error.

Methods: AFI signals $S_1$ and $S_2$ are processed to give $\cos \theta = (Rn - 1)/(n - R)$, where $R = S_2/S_1$ and $n = TR_2/\Delta TR$. The aforementioned AFI approximation leads to an inherent bias, whereas noise produces an uncertainty bias; the net bias is the sum. Noise also leads to an increase in the variance ($\sigma^2$); the relative bias (a measure of accuracy) of random variable $z$ is $\Delta z/z$, and its relative standard deviation (a measure of precision) is $\sigma_z/z$. These can be determined analytically using uncertainty analysis. All work was done using Matlab (8.1 R2013a; MathWorks, Natick, MA). The theoretical results were verified numerically (i.e., via Monte Carlo simulation) by generating $10^5$ instances at each $T_1$ value, adding normally distributed noise, then calculating the mean and standard deviation (SD) accordingly. The maximum theoretical SNR $(= M_0/\sigma_0)$ was set to 1000. The overall error (to roughly 95% confidence) is approximated by the absolute value of the net bias plus two standard deviations.

Results: The relative inherent bias of $\cos \theta$ is $(Rn - 1)/(n - R) - 1$; this error decreases as $T_1$ increases (Figure A). The relative uncertainty bias is given by $(n^2 - 1)(1 + Rn)/[(n - R)^2 (Rn - 1)SNR_{S_1}^2]$, which increases with $T_1$ (Figure B), in contrast to the inherent bias. The relative SD of $\cos \theta$ is given by: $(n - R)(Rn - 1)/[SNR_{S_1} (n^2 - 1)(1 + R^2)^{1/2}]$ (Figure C). Relative overall error estimates are provided in Figure D.

Discussion: The inherent bias of $\cos \theta$ decreases as $\theta$ or $TR_1$ decrease, or as $T_1$ increases since the AFI approximation is $TR_1,2 \ll T_1$. The net bias is the superposition of the inherent and uncertainty biases, so different $(n, TR_1, \theta)$ yield varying proportions. The SD of $\cos \theta$ is essentially the SD of $S_1$ since $n$ and $R$ are constant with respect to $T_1$ for a given $(TR_1, TR_2, \theta)$. The overall error of $\cos \theta$ shows a significant dependence on $(TR_1, \theta)$, and generally increases with $T_1$ (contrary to the inherent bias).

Conclusion: The analysis herein allows one to tailor the AFI parameters to characterize inherent vs. noise-induced errors and minimize overall error. It provides a theoretical and best-case scenario of the error associated with the estimated $\cos \theta$ maps.


Figure: Analytical relative $\cos \theta$ inherent bias (A), Monte Carlo/inherent/uncertainty/net biases (B), standard deviation (C), and overall error (D) versus $T_1$ for $n = 5$ and various combinations of $(TR_1, \theta)$. The y-axis has been capped to 5% in (C) and 10% in (D).