Nuts & Bolts of Advanced Imaging - Parallel Imaging

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Highlights:

- Noise calibration (noise pre-whitening) and SNR scaled reconstruction.
- Non-Cartesian Parallel MRI.
- Regularization in Parallel MRI.

TARGET AUDIENCE: Anybody with an interest in Parallel Imaging (PI) beyond theoretical principles and who would like the tools to implement practical, production-level parallel imaging reconstruction algorithms.

OBJECTIVES: This course aims to describe the effects of colored noise in the parallel MRI [1–3] experiment and how to perform a reconstruction, which takes the noise distribution into consideration. Techniques for SNR scaled reconstruction [4] are described for Cartesian and non-Cartesian imaging. The course will also describe iterative reconstruction algorithms and their regularized forms. Software for all the described reconstruction processes will also be introduced.

PURPOSE: MRI experiments are affected by noise. When a parallel imaging experiment is performed, this noise may be unevenly distributed in the receive channels, e.g. some receive channels may have higher noise levels than others. Furthermore, the noise may be correlated between then channels. If left uncorrected, this can lead to a reduction in image quality (poor SNR). If the noise is pre-whitened, it is possible to maintain unit noise scaling throughout the reconstruction. By doing so, an image can be reconstructed in units of SNR making it easy and intuitive to compare reconstructions. In some reconstructions, such as iterative reconstructions, the SNR reconstruction is not obtained as easily, but we will discuss how one can still obtain images in SNR units using the pseudo-replica (a Monte Carlo simulation) approach. Finally, we will demonstrate how iterative reconstruction can be used to reconstruct data acquired on arbitrary trajectories and we will demonstrate various techniques for performing such reconstructions (including practical software implementations).

METHODS: All MRI experiments collect both signals and some measurement noise:

$$\mathbf{s} = \mathbf{E}\boldsymbol{\rho} + \boldsymbol{\eta} \qquad (1)$$

Where $\mathbf{s}$ is a vector with the measured signal, $\boldsymbol{\rho}$ is the imaged object, $\mathbf{E}$ is the encoding matrix describing the transformation from image space to measured *k*-space (multiplication with coil sensitivities, FFT to *k*-space and sampling) and $\boldsymbol{\eta}$ is the measured noise. The noise is uncorrelated and uniformly distributed between *k*-space locations, but there may be noise level differences between the individual receive channels and the noise may also be correlated between channels. We describe this using the noise covariance matrix, which should be the same for all locations in *k*-space. The noise covariance matrix is an NxN matrix where N is the number of coils, and the $\boldsymbol{i, j}$ entry is defined as:

$$\boldsymbol{\Psi}_{i,j} = <\boldsymbol{\eta}_i, \boldsymbol{\eta}_j^{\mathrm{H}}> \tag{2}$$

Where $<\boldsymbol{\eta}_i, \boldsymbol{\eta}_j^{\mathrm{H}}>$ denotes expected value of the product of the noise in channel i and the complex conjugate of the noise in channel j. If the noise were white and uncorrelated between channels, this matrix would be the identity matrix. In practical experiments this matrix is not identity. As we will demonstrate in this lecture, it can be instructive to inspect this matrix to gain insight into any problems with the receive chain (e.g. broken pre-amplifiers and coil elements). Most reconstructions provide SNR optimal reconstruction results only when the noise is white. To achieve this, we perform a noise pre-whitening step, which creates a set of virtual receive channels where the noise is white, i.e. for a specific location in *k*-space we obtain the noise pre-whitened signal $\boldsymbol{s}_{pw}$ as:

$$\mathbf{s}_{pw} = \mathbf{L}^{-1}\boldsymbol{s} \tag{3}$$

Where

$$\mathbf{L}\mathbf{L}^{\mathrm{H}} = \boldsymbol{\Psi} \tag{4}$$

To perform this procedure, it is necessary to have a set of noise measurements. These noise measurements can (and should) be acquired with every MRI experiments even when parallel imaging is not performed.

Once the data has been noise pre-whitened, it is advantageous to maintain noise scaling throughout the reconstruction. This is done by making sure that every signal processing step is scaled such that $\boldsymbol{\sigma}_{in} = \boldsymbol{\sigma}_{out}$, i.e. the noise standard deviation is the same in each sample (*k*-space or image space) before and after the signal processing step. In practical terms this means that every signal processing step must including an appropriate scaling step. In parallel imaging, the actual unaliasing process introduces a spatially varying noise distribution. Specifically, there will be local noise enhancement due to the g-factor [1]. In this lecture, we will discuss that most Cartesian parallel imaging procedures (SENSE and GRAPPA) can be expressed as phased array combining [5, 6]:

$$\boldsymbol{\rho}(\mathbf{x}, \mathbf{y}) = \sum_{i=0}^{N} \boldsymbol{u}_i(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{a}_i(\boldsymbol{x}, \boldsymbol{y}) \tag{5}$$

Where $\boldsymbol{\rho}(\mathbf{x}, \mathbf{y})$ is the unaliased (SNR optimal) signal at location (x,y), $\boldsymbol{u}_i(\boldsymbol{x}, \boldsymbol{y})$ is the array combining coefficient for coil $\boldsymbol{i}$, at location (x,y), $\boldsymbol{a}_i(\boldsymbol{x}, \boldsymbol{y})$ is the aliased signal at the same location of coil $\boldsymbol{i}$, and N is the number of coils. In other words the reconstructed image is a linear combination of the aliased signals in all channels and the linear combination coefficients (unmixing coefficients $\boldsymbol{u}_i$) vary spatially. Based this expression, the spatially dependent noise amplification (also known as g) can be found as the root sum squares of the coil combining (or unmixing) coefficients:

$$\mathbf{g}(\mathbf{x},\mathbf{y}) = \sqrt{\sum_{i=0}^{N} |\boldsymbol{u}_i(\boldsymbol{x},\boldsymbol{y})|^2} \quad (6)$$

Using this knowledge in combination with the noise scaled signal processing steps, it is possible to obtain an image where the value in each pixel is in units of SNR. The approach for obtaining an SNR scaled image for parallel imaging is outlined in Fig. 1. In this lecture we will explore the utility of such SNR scaled reconstructions when comparing reconstruction algorithms.

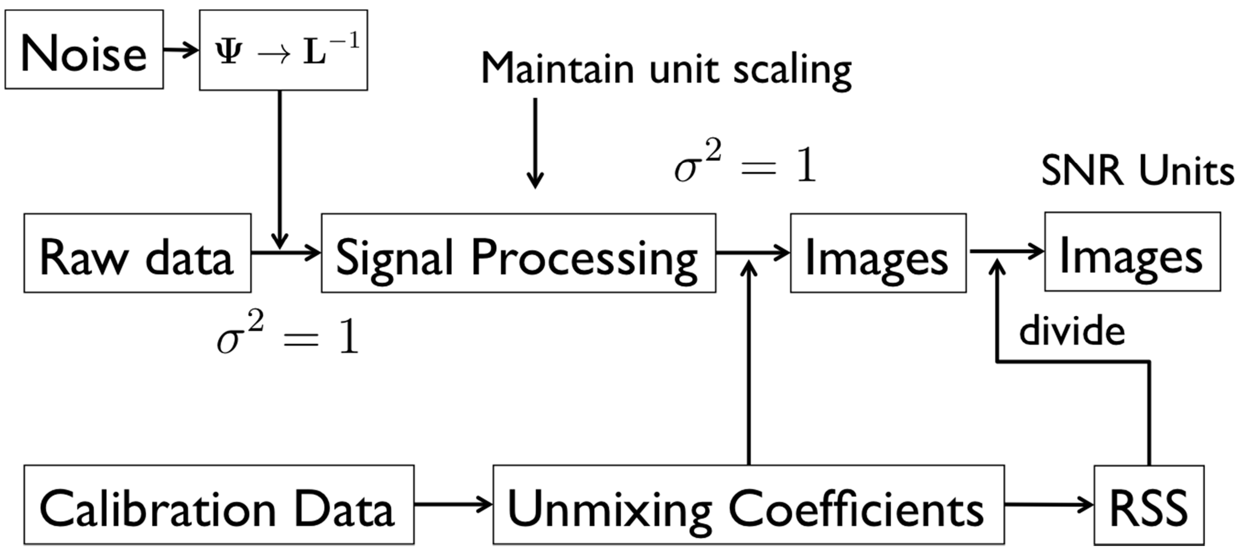

Figure 1. Outline of SNR scaled reconstruction. The reconstruction is preceded by a noise-pre-whitening step. All signal processing steps maintain unit noise variance. The parallel imaging unmixing step introduces a spatially varying noise distribution. This distribution is defined by the g-map. Through a division by the g-map in the final processing step an image in SNR units is obtained.

For some types of reconstruction algorithms, it is not possible to use this SNR scaled approach for evaluating performance. This is the case for algorithms where unmixing coefficients are never explicitly formed (such as iterative reconstruction algorithms) or in cases where parts of the reconstruction code are inaccessible to the user, in which case the scaling may not be known. In such cases, it is possible to use the pseudo-replica method to gain insight into the SNR performance of a given algorithm. With this approach, multiple reconstructions (replicas) are performed and for each replica, noise with a known distribution is added to the original raw data. Ideally the noise should be added with the same distribution (and correlation) as the original experiment. This can be achieved by adding noise after the pre-whitening step described above, in which case the added noise has a noise covariance matrix equal to identity. After repeating the reconstruction a number of times, the noise standard deviation in each pixel can be determined by taking the standard deviation across the replicas and the signal is the mean signal in all replicas. The pseudo replica method provides a straightforward way of evaluating reconstruction algorithms even when some of the reconstruction components are unknown or the algorithm never explicitly forms unmixing coefficients. It is, however, time consuming and not suitable for on-the-fly evaluation of SNR performance on the scanner.

There are some types of parallel imaging problems, where it is not feasible to form unmixing coeffcients explicitly. In these reconstruction algorithms, eq. 1 often has a structure that does not permit a simplification into a set of simpler problems and thus the inversion problem is much more demanding. In those cases, we often use iterative algorithms. In practice, we use an iterative solver to minimize:

$$\|\mathbf{E}\boldsymbol{\rho} - \mathbf{s}\|_2 \tag{7}$$

Often there are additional regularization side constraints, e.g.:

$$\tilde{\boldsymbol{\rho}} = \boldsymbol{arg\ min}\,\{\|\mathbf{E}\boldsymbol{\rho} - \mathbf{s}\|_2 + \boldsymbol{\lambda}\|\mathbf{T}\boldsymbol{\rho}\|_2\} \tag{7}$$

Where $\mathbf{T}$ is some linear transform of the solution and $\boldsymbol{\lambda}$ is a tunable regularization parameter used to control the tradeoff between data consistency and regularization. In this lecture we will review basic techniques and software for solving such problems and demonstrate how these techniques form the basis for iterative SENSE [7] and the SPIRiT [8] algorithm.

CONCLUSION:

This lecture focuses on how to deal appropriately with noise in the acquired data. We will review techniques for noise pre-whitening and ways to arrive at SNR scaled images. The lecture also introduces iterative parallel imaging reconstruction with some practical implementations

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