Maxwell's Equations and Electromagnetic Field Simulations

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Magnetic Resonance is typically accomplished with application of both a static (or DC) magnetic field, B_0, and a radiofrequency (RF) magnetic field, B_1. To additionally accomplish imaging, switched or audiofrequency gradient fields, G_x, G_y, and G_z are typically applied. While the interactions of these fields with the net nuclear magnetization vector at a specific location as required for NMR and MRI can often be adequately described with the Bloch equation(s), many general aspects of signal, noise, artifacts, and safety cannot be understood without consulting the Maxwell Equations, given below in differential (left) and integral (right), time-dependent form.

Differential | Integral

$$\nabla \bullet \varepsilon\mathbf{E} = \rho \qquad \oiint_s \varepsilon\mathbf{E} \bullet d\mathbf{s} = \iiint_V \rho dV \tag{1}$$

$$\nabla \bullet \mathbf{B} = 0 \qquad \oiint_s \mathbf{B} \bullet d\mathbf{s} = 0 \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \qquad \oint_l \mathbf{E} \bullet d\mathbf{l} = -\frac{\partial}{\partial t}\iint_s \mathbf{B} \bullet d\mathbf{s} \tag{3}$$

$$\nabla \times \mathbf{B} = \mu\left(\sigma\mathbf{E} + \frac{\partial}{\partial t}\varepsilon\mathbf{E}\right) \qquad \oint_l \mathbf{B} \bullet d\mathbf{l} = \mu\left(\iint_s \sigma\mathbf{E} \bullet d\mathbf{s} + \frac{\partial}{\partial t}\iint_s \varepsilon\mathbf{E} \bullet d\mathbf{s}\right) \tag{4}$$

Here **E** and **B** are the electric field and magnetic flux density vector fields, ε is electrical permittivity, ρ is charge density, μ is magnetic susceptibility, and σ is electrical conductivity. To express these in time harmonic form, as is often done for analysis of RF fields at a single frequency, the partial derivative with respect to time ($\partial/\partial t$) is replaced with $j\omega$ where j is the imaginary unit and ω is the radial frequency of the time-varying field (especially the Lamour precession frequency of the nucleus and B_0 strength of interest).

All of these equations have relevance to some aspect of MRI. For example, Gauss's Law for Magnetism (Eq. 2) coupled with the relation $B=\mu H$ can be used to describe subtle distortions of B_0 related to miniscule differences in μ between air and tissue (or between different tissues), which can result in notable undesired artifacts (or desired contrast) in MRI (1-4). Gauss's Law for electric fields (Eq. 1) can be used to describe "conservative" electric fields arising from charge density on coil conductors (5, 6). As indicated with Faraday's Law (Eq. 3), a time-varying magnetic field can induce electric fields (and corresponding electric currents) in a nearby conductor. This can result in detectable signal during signal reception if the conductor is a receive coil and the time-varying field is caused by the rotating net nuclear magnetization vector (7), or in adverse heating during nuclear excitation if the conductor is human tissue and the time-varying field is the transmitted B_1 field (8-11). Ampere's Law (Equation 4 minus the right-most term) describes how, at low frequencies or over electrically small distances, an electrical current density, $J=\sigma E$, can produce a magnetic field throughout space (12, 13), and Maxwell's correction to this, the "displacement current term" on the right shows how a time-varying electric field can also induce a magnetic field, which is increasingly important in MR at increasingly high B_0 strengths and correspondingly higher B_1 frequencies and shorter RF wavelengths (14-16).

General understanding of trends and relations can be understood with some analytically-based representations of Maxwell's Equations (7, 8, 17, 18). For highly accurate understanding of the behavior of electromagnetic fields in specific complex geometries (such as specific coil designs or the human body) requires numerical simulation (1, 2, 3, 9, 10, 14-16, 19-22).

In this talk we will discuss several examples of how Maxwell's equations are key to understanding signal, noise, artifacts, and safety in MRI. We will discuss examples of how analytically-based calculations can yield valuable insights, and how numerical simulations can produce specific information. Strengths and weaknesses of different simulation approaches will be compared. Attendees should have an increased appreciation for the relevance of Maxwell's equations to all aspects of MRI, and an improved understanding of how electromagnetic fields can be simulated for analysis of signal, noise, artifacts, and safety in MRI.

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