Nuts & Bolts of Advanced Imaging: Parallel Imaging Reconstruction Design

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Highlights

- **Most non-iterative reconstruction methods can be separated into a calibration step that generates channel combination and unaliasing coefficients followed by a step that applies these coefficients to create reconstructed images.**
- **Maps of image shading, noise amplification and aliasing energy can be created to better assess the image quality achieved by a reconstruction pipeline.**
- **Complete characterization of coil sensitivities is not necessary to perform parallel imaging.**

TARGET AUDIENCE: Imaging researchers and students who would like to learn more about the practical issues involved in Parallel Imaging reconstruction.

OBJECTIVES: This course will provide attendees with background theory and software tools that will make it easier for them to integrate parallel imaging into their projects and applications. This course will also equip attendees with an understanding of the design tradeoffs involved in choosing a parallel imaging strategy. The software tools and exercises part of this course will be available at http://gadgetron.sourceforge.net/sunrise.

PURPOSE: Multi-channel receiver coil arrays are ubiquitous on modern scanners and even researchers and students whose research is not focused on Parallel Imaging often need to reconstruct multi-channel data sets. Whether using existing reconstruction routines or developing custom routines, an understanding of the issues involved with Parallel Imaging is necessary to use these routines effectively. The lectures and online software tools and exercises that are part of this course provide an opportunity to better understand the functionality and limitations of the components that make up a Parallel Imaging reconstruction pipeline.

METHODS:

A great deal of insight into the practical considerations involved with Parallel Imaging can be obtained by manipulating the mathematical model used to represent multi-receiver channel imaging. The acquired data can be decomposed into a signal and noise component (see Table 1 for details on mathematical notation):

$$d_j(k_x, k_y) = M_j(k_x, k_y) + N_j(k_x, k_y)\,, \qquad (1)$$

where the signal component combines the transverse magnetization with coil sensitivity information and Fourier (gradient) encoding:

$$M_j(k_x, k_y) = \iint s_j(x, y)\, e^{i2\pi(k_x x + k_y y)} m(x, y) dxdy, \qquad (2)$$

and the noise component is a Gaussian random variable as described in Table 1.

Channel Combination and Uniform Sensitivity

When full Fourier encoding is used, a Fourier Transform applied independently to each channel transforms the signal into images of the magnetization shaded by the sensitivities of the receiver coils:

$$m_j(x,y) = s_j(x,y)\, m(x,y). \tag{3}$$

Creating a single composite image can be accomplished by performing an independent linear combination of the channels at each spatial location:

$$\widehat{m}(x,y) = \sum_j c_j(x,y) m_j(x,y), \tag{4}$$

where $c_j(x,y)$ are channel combination coefficients that are determined as part of the image reconstruction. From a signal perspective, we would like our composite image $\widehat{m}(x,y)$ to have uniform sensitivity. This can be accomplished by choosing channel combination coefficients that satisfy

$$\sum_j c_j(x,y) s_j(x,y) = 1. \tag{5}$$

When Eq. 5 is not satisfied, the reconstructed image will not have uniform sensitivity and this can result in a visible shading artifact, where the image is brighter in one region compared to another. Most commonly, this manifests as images being bright at locations near the coil elements and it can sometimes be hard to differentiate this receiver coil sensitivity artifact from shading due to spatial variations in transmit efficiency.

Channel Combination and Noise

The noise in the data can be separately followed through the Fourier Transform operation and results in noise at each pixel position that is similarly correlated across channels. We can denote this noise in the channel images as Gaussian random variables $n_j(x,y)$. When channel combination occurs, the noise in the composite image can be expressed as the resulting random variable:

$$n(x,y) = \sum_j c_j(x,y) n_j(x,y). \tag{6}$$

The standard deviation of $n(x,y)$ is a function of the channel combination coefficients. Choosing channel combination coefficients that satisfy Eq. 5 generally results in the noise standard deviation changing across spatial locations. The means that the noise level in images formed form multi-channel data varies across the image, making the estimation of the image signal-to-noise ratio (SNR) more difficult. Specifically, estimating the noise level from a region without signal can give misleading results, since the noise level could be quite different in a region that contains signal. A useful way of visualizing noise amplification is as a map: plotting the standard deviation of $n(x,y)$ as an image gives a “noise amplification map” for a reconstruction.

Optimal Channel Combination Coefficients

Unless otherwise specified, optimal channel combination coefficients usually refers to channel combination coefficients that are optimal in terms of SNR. Optimal channel combination coefficients can be computed independently at each pixel location. With N coils, we can concatenate the coil sensitivity values $s_j(x,y)$ at a given pixel location into a vector, **s**, of length N (where each element is the sensitivity at the given pixel for one of the channels). The channel combination coefficients at the same pixel location can be similarly concatenated into a length N vector, **c**. For noise correlation across channels can be expressed as an NxN noise correlation matrix, Ψ. Ψ does not change with pixel location. Optimal SNR can be achieved by choosing channel combination coefficients that satisfy

$$\boldsymbol{c} = \gamma \boldsymbol{s}^H \boldsymbol{\Psi}^{-1}, \tag{7}$$

where the H superscript indicates taking the Hermitian conjugate [1]. γ is a scalar value that can be chosen without impacting SNR, since increasing the magnitude of γ increases both the signal and noise equally. A unique set of the channel combination coefficients results from satisfying both Eq. 5 and eq. 7, which results in $\gamma = 1/\boldsymbol{s}^H \boldsymbol{\Psi}^{-1} \boldsymbol{s}$. When the noise correlation matrix is identity (e.g. because a pre whitening step has been applied), the optimal channel combination coefficients can be expressed as

$$c_j(x, y) = \gamma s_j^*(x, y) \tag{8}$$

Absolute and Relative Coils Sensitivities

To generate SNR optimal channel combination coefficients, it is necessary to know the relative sensitivity between the channels at each pixel location, but it is not necessary to know how the sensitivity of any channel changes with position, in any absolute sense. To illustrate this difference, consider one method for estimating channel sensitivity information from a set of calibration images acquired on a multi-channel array. Disregarding noise and the resolution-dependent point-spread function, we can express the calibration images as the channel sensitivities multiplied by the transverse magnetization in the calibration scan, as in Eq. 3. A set of relative coil sensitivities [2] can be produced by dividing each of these images by the square-root sum-of-squares of the calibration images:

$$\hat{s}_j(x, y) = \frac{m_j(x, y)}{\sqrt{\sum_{j'} \left| m_{j'}(x, y) \right|^2}} = \frac{s_j(x, y)}{\sqrt{\sum_{j'} \left| s_{j'}(x, y) \right|^2}} e^{i \angle m(x,y)}. \tag{9}$$

Using the relative coil sensitivities computed in Eq. 9, or by some similar means, will result in SNR optimal channel combination, but image shading could remain because Eq. 5 may not be satisfied. To satisfy Eq. 5, it is necessary to have additional information on how the sensitivity of at least one of the channels varies in space, in an absolute sense. One method for achieving this is to collect additional calibration information from a uniformly sensitive coil. However, even without absolute coil sensitivity information, both SNR optimal channel combination and Parallel Imaging can be performed if image shading can be tolerated.

The reader will also notice that Eq. 9 leaves a residual phase from the calibration magnetization. Designing a calibration scan to minimize the phase in the transverse magnetization is one way to deal with this complication. If this is not possible (e.g. if the calibration scan is embedded in an out-of-phase acquisition), this phase will propagate to the channel-combined composite image. If a magnitude image is taken, this added phase will disappear. Because most phase sensitive applications like temperature mapping or chemical shift encoded imaging rely on the relative phase between two or more acquisitions, as long as the same set of relative coil sensitivities is used for all acquisitions, the phase from the calibration magnetization will cancel out.

Uniform Undersampling

Uniform undersampling is a very common way to take advantage of Parallel Imaging. One very attractive feature of uniform undersampling is that the procedure used with unaccelerated imaging can be repeated. Specifically, we can apply the Fourier Transform independently to each channel and then use independent linear combinations at each pixel location to combine the data into a composite image. However, the channel combination coefficients are replaced with coefficients that perform both a channel combination and unaliasing operation. Thus, we can rewrite Eq. 4 for the accelerated case as

$$\widehat{m}(x,y) = \sum_j u_j(x,y) a_j(x,y) \tag{10}$$

Where $u_j(x,y)$ are a set of unaliasing and channel combination coefficients and $a_j(x,y)$ are aliased images that result from taking the Fourier Transform of uniformly undersampled data.

There are many different ways that the coefficients, $u_j(x,y)$ can be generated. The procedure for generating these coefficients is often called the calibration phase of a parallel imaging reconstruction. In the SENSE method [3], these coefficients are generated directly by solving a least norm problem composed using the channel sensitivities. The data driven calibration used in the GRAPPA [4] method can also be used to generate these coefficients. To do this, k-space GRAPPA coefficients must be converted into image space and combined with channel combination coefficients.

Aliasing Energy

Parallel Imaging acceleration introduces the potential for aliasing artifacts. If we know the true coil sensitivity functions, we can evaluate a set of unaliasing coefficients to create an "aliasing energy map" that indicates which image locations are prone to have aliasing.

G-Factor

Equation 6 can be extended to the accelerated case by replacing the channel combination coefficients with the unaliasing and channel combination coefficients $a_j(x,y)$. These new coefficient will alter the noise amplification map, generally leading to increased noise in the final image. The g-factor isolates the effect of reduced gradient encoding on the noise amplification map by dividing the accelerated noise amplification map by the unaccelerated noise amplification map and correcting for the difference in scan time.

DISCUSSION & CONCLUSION:

Parallel Imaging introduces image shading, spatially varying noise and aliasing artifacts as three issues that must be addressed during image reconstruction and analysis. In the case of uniform undersampling, maps can be created to visualize each of these issues over the image. Creation of these maps relies on access to true coil sensitivity information.

$m(x, y)$	Transverse magnetization signal that is being imaged
$d_j(k_x, k_y)$	Datum acquired on channel with index j. k_x and k_y indicate the gradient encoding at the time the datum is acquired. More completely, the k-space location can be written as a function of time, $k_x(t), k_y(t)$. I drop the parameter t in following expressions in favor of more compact notation.
$M_j(k_x, k_y)$	Signal component of datum acquired on channel with index j at k-space location k_x, k_y.
$N_j(k_x, k_y)$	Noise on channel with index j at k-space location k_x, k_y. For trajectories that resample the same k-space location, k_x, k_y should be replaced with t. Noise is modelled as Gaussian random variable that is independent and identically distributed along time (k-space location), but can be correlated and not identical across channels.
$s_j(x, y)$	Coil sensitivity; j indexes the coil array elements and x,y give the spatial position
$e^{i2\pi(k_x x + k_y y)}$	Gradient, or Fourier, encoding; k_x, k_y give the k-space location and x,y give the spatial position.
$\widehat{m}(x, y)$	An estimate of the transverse magnetization signal

Table 1: Mathematical Notation

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