Fast Method for Parametric System Identification of Gradient Systems

Yu-Chun Chang[1,2], Martin Eschelbach[1], Nikolai Avdievitch[1], Klaus Scheffler[1], and Anke Henning[1,3]

[1]*Max Planck Institute of Biological Cybernetics, Tuebingen, Baden-Wuerttemburg, Germany,* [2]*Graduate School of Neural & Behavioural Sciences, University of Tuebingen, Tuebingen, Baden-Wuerttemburg, Germany,* [3]*Institute for Biomedical Engineering, University and ETH Zurich, Switzerland*

Introduction: The characterisation of the gradient system impulse response can be used as prior knowledge in image reconstruction, parallel TX RF pulse design and for pre-emphasis to correct for imperfections in the B0 field that may arise. The gradient system has previously been characterised using field monitoring in combination with frequency domain approaches [1-2]. Although this method can model the system quite accurately non-parametric models result, which means that they do not have an analytic form. A field monitoring method for system identification of a gradient system using a parametric model is demonstrated. Parametric models allow analogue pre-emphasis circuits to be more easily designed and also allow classical design methods for closed-loop feedback systems to be used, which is particularly advantageous for multivariable systems such as gradient and B0 shim systems. Furthermore the proposed method requires only one measurement for each gradient thus being faster than previously described frequency domain approaches based on repeated measurements with chirp signal inputs [2] while still obtaining a high prediction accuracy.

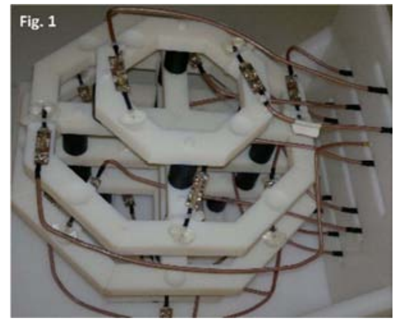

Fig. 1

Method: The magnetic field was monitored using a custom-built 9.4T field camera that consisted of sixteen 1H NMR probes mounted on a 250mm-diameter sphere (fig. 1) similar to the one described in [3]. The NMR probes were tuned and matched (399.72MHz, 50Ω) and were also decoupled using cable traps (more than 42dB isolation between any two probes). The probes were susceptibility matched [3] and the water samples were doped with $CuSO_4$ reducing the T1 and T2 to approximately 80ms. The probes were operated in transmit/receive mode using a home-built 16 channel interface (-50 dB isolation, 0.2 dB insertion loss per channel) and simultaneously excited by rectangular RF pulses of duration 0.5ms. Since gradient fields cause de-phasing to occur relatively quickly, the TR between the pulses was 15ms. The measurements were performed on a whole body 9.4T Siemens Magnetom scanner (Erlangen, Germany).

For any linear time-invariant system, the response to any input can be predicted using the impulse response function (IRF) in the Laplace domain: $out(s) = irf(s) * in(s)$. To determine the impulse response irf(s) gradient trapezoids with a slew rate of 44mT/m/ms and an amplitude of 10mT/m were used as inputs in(s). The FID signals were sampled at 300kHz. Related measurements were performed for X, Y and Z gradients. The phase coefficients were extracted from the FID phase measurements by fitting spherical harmonic functions to the acquired 16-channel data. Only zero- and first-order harmonics were used since only the gradient system was characterised (higher-order harmonics up to the 3rd order can be used for shim systems characterisation with the current setup). Fitting of the spherical harmonic functions require the position of the probes to be known. This was calculated from the derivative of the phases after the gradient fields had settled and reached steady-state. Once the phase coefficients were calculated, the field changes were obtained by taking their derivative.

Table 1: Gradient system parameters

System response	A	a	ω
X-X	0.98	54.4e3	27.2e3
Y-Y	1.00	42.8e3	35.8e3
Z-Z	0.995	42.4e3	34.2e3
X-F0	5.23	31.4e3	N/A
Y-F0	-1.78	27.8e3	N/A

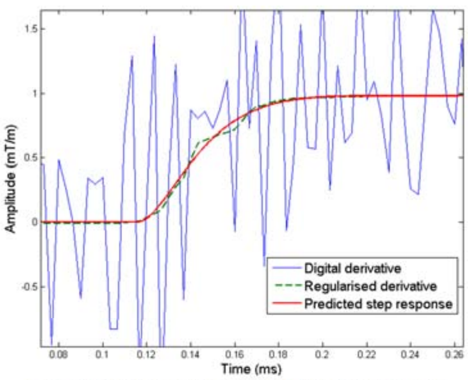

Fig. 2: The derivative of the field coefficient during the gradient ramp (with slew rate 44mT/m/ms) period of the x-gradient. The gradient ramp was started at 0.12ms.
The digital derivative uses the finite different between two adjacent samples as the derivative.

The gradient system was modelled as a multivariable system where the order of each transfer function was assumed to be at most second-order; so the equation for the IRF is $Ae^{-at}\sin(\omega t)$ for second-order and Ae^{-at} for first-order. The slew rate is not fast enough to approximate a step response and modelling the output data using a step input would not be sufficiently accurate. As an alternative the ramp up of the gradient was used to model the fast transient effects. The input was assumed to be a perfect ramp with a gradient of 44mT/m/ms. Therefore by differentiating the field coefficient would give the step response to a step input of amplitude 44mT/m. However the derivative of the ramp is very noisy if a digital derivative is performed. To obtain smooth derivatives of the output data, a total-variation regularization algorithm was used with a regularisation parameter α = 5e-6 [4] (fig. 2). The parameters to the second-order transfer functions were found by minimising the error between the measured and predicted output.

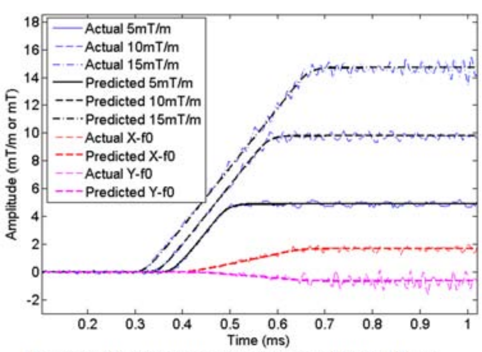

Fig. 3: Predicted and actual response of the x-gradient for different amplitudes. The cross-terms from the x- and y-gradients (10mT/m) to the zero-order term (mT) are also shown.
The input are gradients with different amplitudes (see figure legend) and a slew rate of 44mT/m/ms.

Results and Discussion: The transfer function parameters for the gradient self-terms are shown in table 1.

The gradient system had negligible cross-term effects which was expected because individual gradient coils are well decoupled due to the shielding therefore the gradient system can be treated as three independent single-input single-output systems. However there are cross-term effects from the x- and y-gradient to the zero-order term. These effects were modelled with a first-order system and the parameter values are given in table 1. To verify the results, the predicted output from the applied input (10mT/m with 44mT/m/ms slew rate) was compared to the actual output. This was also done for gradient input amplitudes of 5mT/m and 15mT/m (fig. 3).

Although previous system identification methods [1] can characterise the high frequency responses, these effects decay very quickly and thus the slower low frequency responses are the dominant components. So although a second-order parameterisation captures the lower frequency components, this simplification is valid (fig. 3).

This method uses the ramp of the gradient which is normally very short and can therefore only measure fast dynamics. To obtain an approximation of long term effects, such as eddy current effects, the gradient input can be approximated as a step response.

Conclusion: Identification of the gradient system using parametric models can be performed quickly and efficiently. This accuracy of the models can be increased by increasing the order of the model transfer functions. This method can also be used to characterise shim systems using higher order spherical harmonics and obtain a multivariable transfer function which can be used in pre-emphasis or controller design.

References

[1] S. Vannesjo et al., MRM 2013,69:583-593; [2] S. Vannesjo et al, MRM 2013, doi:10:1002/mrm.24934; [3] C. Barmet et al., MRM 2008; 60:187-197; [4] R. Chartrand, ISRN Applied Mathematics 2011, article ID 164564.