Temperature simulations for the inverse boundary element gradient coil design method

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Target Audience Gradient coil designers

Purpose To model the temperature distribution inside gradient coils of arbitrary geometry. The maximum temperature in a gradient coil is a limiting factor in its maximum duty cycle. In this work we develop a model of temperature for the inverse boundary element[1] (IBEM) coil design process. Simulation allows the temperature distribution to be estimated for numerous coil designs. Accurate temperature simulation relies on experimental tuning of the simulation parameters[2] for significantly different coil materials and structures.

Methods The temperature above ambient, T^*, can be modelled by a heat equation[2] in which the rate of change of temperature is dampened by the heat capacity of the object, the dissipation of heat throughout the object is governed by the Laplacian of the temperature and heat is lost from the object by a cooling term proportional to T^*. An ohmic heating term is included for the power dissipated as heat energy due to large current densities, $\mathbf{j}$.

$$c_h \rho_d \frac{\partial T^*}{\partial t} = k_c \nabla^2 T^* + \frac{\rho_r}{w^2}\mathbf{j}\cdot\mathbf{j} - \frac{h_t}{w}T^*, \qquad (1)$$

where c_h is the specific heat capacity, ρ_d is the mass density, k_c is the thermal conductivity, ρ_r is the electrical resistivity, w is the coil layer thickness and h_t is the cooling coefficient. Careful choice of these parameters was shown to be critical for correct prediction of measured temperatures[3].

Equation (1) is a partial differential equation that must be discretised. In boundary element method (BEM) based coil design techniques, the surface is often modelled as flat triangular elements in which uniform $\mathbf{j}$ can flow. The coil design is usually parameterised in terms of stream-function of the current density, which is a piecewise-linear scalar function on the mesh defined by a set of values, ψ, at the mesh nodes. The discrete current density is defined in each triangle, $\mathbf{j} = J\psi$, which here were interpolated back to the mesh nodes. In this work we use the conformal discrete Laplacian operator, L, in place of its continuous counterpart, ∇^2, where $L = D - W$, $D = \mathrm{diag}_i(\sum_j W_{i,j})$ and $W_{i,j} = \cot(\alpha_{i,j}) + \cot(\beta_{i,j})$. $\alpha_{i,j}$ and $\beta_{i,j}$ are the angles opposite the edge connecting nodes i and j.

The steady-state temperature, T^*_{ss}, can be solved for with a matrix equation since the time differential becomes zero:

$$T^*_{ss} = -\frac{\rho_r I^2}{w}\left(w k_c L - H_t\right)^{-1}\left(J\tilde{\psi}\right)^2 \qquad (2)$$

where $I^2(J\tilde{\psi})^2 = \mathbf{j}\cdot\mathbf{j}\,I$ is the current and $\tilde{\psi}$ is the normalised ψ. Alternatively, an Euler time stepping scheme can be used to discretise time: $\partial T^*/\partial t \to (T^{(\ell+1)} - T^{(\ell)})/\tau$.

Results Figures 1 and 2 demonstrate the method using the parameters c_h, ρ_d, k_c, ρ_r, w, h_t with values 385 J kg^{-1} K^{-1}, 8960 kg m^{-3}, 401 W m^{-1} K^{-1}, 1.68×10^{-8} Ω m, 4 mm, 153.5 W m^{-2} K^{-1} respectively. The coil in Fig. 1 was designed with the IBEM[1] by minimising the resistance. The maximum[4] and sparsity[5] of $|j|$ was minimised for the coil in Fig. 2. Five thousand time points were simulated in ~ 2 s. The current was simulated to be on for 250 s and off for a further 250 s.

Discussion These simulations are sensitive to the thermal parameters and should be approximately matched to experimental results, as was previously reported[3]. It is expected that this forward modelling can be inverted to provide a method of designing minimum maximum temperature coils[6]. It should also be possible to include other partial differential equations of similar form into the BEM, such as vibrational dynamics. Our simulations show heating over a few minutes because the simulation parameters match single layer prototypes used for validation in previous work.

Conclusion We have demonstrated thermal simulations for the IBEM gradient coil design. The method should be compared to experiment and then used to predict temperatures of new coil designs.

References [1]S Pissanetzky, *Meas Sci Technol* (1992) **3** p667, [2]P While et al., *Concepts Magn Reson B* (2010) **37B** p146, [3]M S Poole et al., *MRM* (2012) **68** p639, [4]M S Poole et al., *J Phys D* (2010) **43** 095001 (13pp), [5]M S Poole and N J Shah, *Proc ESMRMB* (2013) **30** p406, [6]P While et al., *MRM* (2013), **70**, p584, [7]dx.doi.org/10.6084/m9.figshare.847301, [8]dx.doi.org/10.6084/m9.figshare.847302

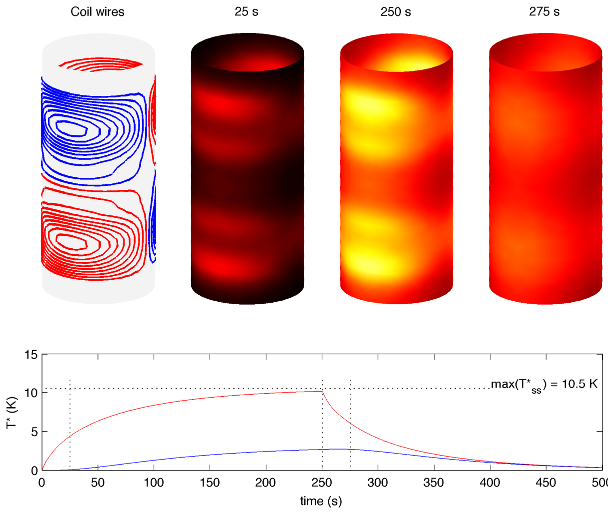

Figure 1. Heating and cooling of a small-scale prototype X gradient coil. Top left shows the wire centres of the coil, where red wires have reversed current sense with respect to blue. Top right are 3 temperature distributions plotted over the surface just after I is switched on, close to thermal equilibrium and shortly after I is switched off. Bottom graph shows the maximum (red) and minimum (blue) surface temperatures over time. Black dotted lines show the times of the temperature plots above and the thermal equilibrium temperature using Eq. (2). An animation is available online[7].

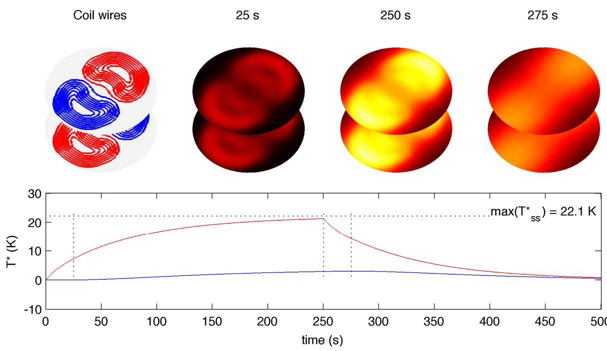

Figure 2. Heating and cooling of a prototype X gradient coil for a portable permanent magnet system. An animation is available online[8].