Integral Equations Based Modeling Approach to Dielectric Shimming

Wyger Brink[1] and Andrew Webb[1]
[1]Radiology, Leiden University Medical Center, Leiden, Zuid-Holland, Netherlands

Target audience: Researchers and RF engineers interested in dielectric shimming.

Purpose: Dielectric shimming offers many degrees of freedom such as the permittivity, geometry and positioning of the high dielectric material. The procedure of optimizing dielectric shims typically relies on full-wave numerical simulations since the current distribution induced in the dielectric shim is required and not known in advance [1]. In this work, we present a numerical approach which reduces the problem complexity when modeling the effects of a dielectric shim using an integral equation based approach.

Methods: We consider here the problem of describing the RF field perturbation caused by dielectric shims with a relative permittivity of ε_r = 300, corresponding to an aqueous suspension of barium titanate, designed for cardiac imaging as described in [2]. The RF field is formulated using an integral equations approach, in which we incorporate the inhomogeneous body model, provided by the Virtual Family dataset [3], explicitly into the background through the Green's function [4]. This reduces the contrasting material parameter to the dielectric shims, meaning that interactions within the body itself are automatically resolved.

A two-dimensional numerical experiment was performed using the iterative conjugate gradient method using a 5-mm spatial grid [5]. The body coil was modeled using a circular array of 16 electric current line sources driven at 128 MHz positioned at a radius of 30 cm. A [0,2π] phase evolution was applied across its circumference to simulate the fundamental quadrature mode excitation. All routines were implemented in Matlab (2012b, Mathworks, Natick, MA).

Let $\boldsymbol{E}^{\mathrm{b}}$ and $B_1^{+,\mathrm{b}}$ denote the electric and forward polarized magnetic fields present in the background medium, i.e. without any dielectric shim. The total electric field $\boldsymbol{E}$ can then be expressed through an integral equation as follows:

$$\boldsymbol{E} = \boldsymbol{E}^{\mathrm{b}} + \int_{\boldsymbol{r}'\in \boldsymbol{D}} g^{\mathrm{b}}(\boldsymbol{r}-\boldsymbol{r}')\chi_{\mathrm{e}}(\boldsymbol{r}')\boldsymbol{E}(\boldsymbol{r}')\mathrm{d}V \qquad (1)$$

in which g^{b} denotes the background Green's function, which now needs to be evaluated numerically in advance since no analytical formulation can be applied. The electric susceptibility $\chi_{\mathrm{e}} = \varepsilon_r - 1$ is now only related to the dielectric shim. Once the electric interactions within the dielectric shim are resolved, the total B_1^+ field follows as

$$B_1^+ = B_1^{+,\mathrm{b}} + \int_{\boldsymbol{r}'\in \boldsymbol{D}} \nabla g^{+,\mathrm{b}}(\boldsymbol{r}-\boldsymbol{r}')\times\chi_{\mathrm{e}}(\boldsymbol{r}')\boldsymbol{E}(\boldsymbol{r}')\mathrm{d}V \ . \qquad (2)$$

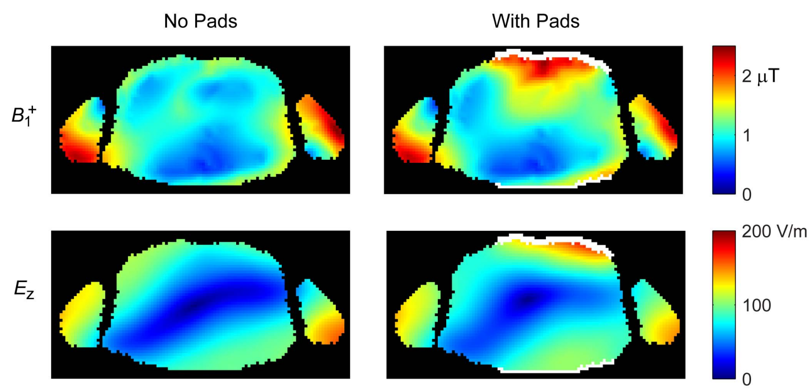

Figure 1. B_1^+ (top) and E_z-field magnitude distribution simulated in 2D without (left) and with (right) dielectric pads (indicated in white) using the proposed method.

Results: Figure 1 shows the simulated B_1^+ and E_z-fields without and with pads, using the proposed method. Figure 2 shows the convergence of both conjugate gradient schemes, one with the standard free space Green's function and the second using the proposed method with inhomogeneous background. As can be seen from the convergence speed, the proposed method requires many fewer iterations.

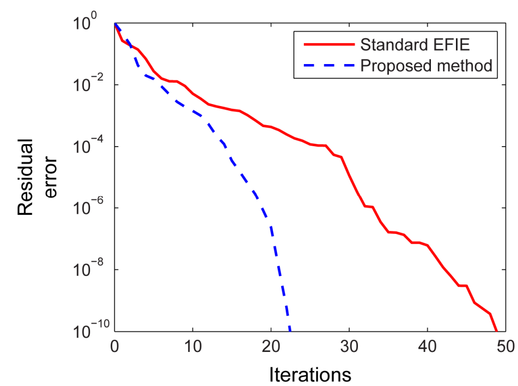

Figure 2. Convergence of the conjugate gradient schemes based on a standard free space Green's function and the proposed inhomogeneous background Green's function.

Discussion: As wave-like effects become more dominant in the behavior of a dielectric shim, one needs to consider the full-wave description of the RF fields for modeling their effect. The method presented here provides both an intuitive formulation as well as an efficient numerical method for computing the effect of a dielectric shim, at the expense of additional computational overhead. The second part of eqs. (1) and (2) represents the 'secondary field' as discussed in literature before [1, 6-7]. Since tissue loading is explicitly taken into account through the Green's function, the problem is reduced to solving only for the (unknown) electromagnetic interactions within the dielectric shim thereby reducing the problem complexity.

Conclusion: A numerical framework based on the integral equations method with an inhomogeneous background model provides an efficient and intuitive approach for formulating the effect of a dielectric shim on the RF field.

References: [1] Brink et al., *Proc. ISMRM 2013*, 4375; [2] Brink et al., *MRM 2013*, doi: 10.1002/mrm.24778; [3] Christ et al., *Phys Med Biol 2010*, 55:N23–N38; [4] Brink et al., *Proc. ICEAA 2013*, 528–531; [5] Zwamborn et al., *IEEE Trans. Microw. Theory Techn. 1991*, 39:953–960; [6] Yang et al., *JMRI 2006*, 24:197–202; [7] Luo et al., *MRM 2013*, 70:269–75.