Hybrid inverse/FDTD computational electromagnetic methodology and its potential application in MR RF field design

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Audience: For the peers who are interested in development of new computational methodology for RF field design at high field MRI.

Introduction: The inverse method in computational electromagnetic calculation has its particular advantage, that is, the source electromagnetic field can be specially designed from the targeted field, which can be designed exactly [1]. Using inverse method in MR RF field design can obtain flexible, goal-oriented, and optimized source current, on which to base RF coil design. However, it is difficult to directly use inverse method for the human body in the high-field MR environment, where the complicated coil-tissue interaction must be considered because of the RF wavelength being comparable or less than the dimensions of the human body. Here we report initial results of utilizing the Finite Difference Time Domain (FDTD) method for inverse design and validation in a limited case (small target region) with consideration of the presence of the human body [2]. A Huygens' equivalent surface was established to integrate the FDTD methods into the inverse methods. An example of the potential application using the new hybrid method is presented.

Materials and Methods: 1. *Setup of target magnetic fields*. In principle, any shape, localization, and RF field distribution of desired target field can be set first as the goal, based on which the source electromagnetic field is deduced accordingly. Here, a target field with desired distribution inside a female pelvis region was used (Fig 1). 2. *Establishment of a Huygens' equivalent surface*. A cylindrical-shape Huygens' equivalent surface was established, with the radius of $c = 20cm$ and length of $2b = 40cm$, used as the bridge between the FDTD method and the inverse method. The FDTD solver was used to calculate the electromagnetic field distribution on the established Huygens' equivalent surface, where the target field was regarded as the source of the FDTD solver and the human body was loaded into the solver, the perfectly matched layers [2] were set just outside of the Huygens' equivalent surface to avoid large regions calculation. 3 *Determination of the current density distribution on the electromagnetic field source*. The current density distribution of the electromagnetic field source was determined according to the electromagnetic field distribution on the established Huygens' equivalent surface using inverse method [3]. Let $F = \iint [(H_x - H_{tx})^2 + (H_x - H_{tx})^2 + (H_x - H_{tx})^2] ds = 0$ (1), where H_{tx}, H_{ty}, and H_{tz} are the x, y, and z components of the magnetic fields on the Huygens' equivalent surface; H_x, H_y, and H_z, are the x, y, and z components of the magnetic fields on the Huygens' equivalent surface induced by the source current density. Three orthogonal components instead of two orthogonal components of the magnetic fields were used in Eq. (1), in contrast to the previously reported inverse methods otherwise only x and y components were considered [1,3]. This is due to the fact that the complex field-tissue interactions can produce three orthogonal components of magnetic fields on the Huygens' equivalent surface. The source current density distribution on the source surface (also supposed to be cylindrical shape, outside the Huygens' equivalent surface) can be expressed as $\vec{J}(\vec{r'}) = [j_z(\phi', z')\hat{z} + j_\phi(\phi', z')\hat{\phi}]\delta(\rho' - a)$, where $j_z(\phi', z')$ and $j_\phi(\phi', z')$ are the z and ϕ components of $\vec{J}(\vec{r'})$, respectively. The z' component is within the region of $-L \le z' \le L$ and should vanish at the boundaries $\pm L$, that is $j_z(\phi', z') = 0$. Fourier's series form of $j_z(\phi', z')$ and $j_\phi(\phi', z')$ were used here. For example, $j_z(\phi', z') = \sum_{m=1}^{M} \sum_{n=1}^{N} (1/2L) sinn\pi(z' + L)(a_{mn} cosm\phi' + b_{mn} sinm\phi')$, where a_{mn}, b_{mn}, etc. are the coefficients. The solutions of F was minimized with respect to the coefficients to obtain the optimized solutions. For instance, for the coefficient a_{ij}, the function F was set to satisfy $\frac{\partial F}{\partial a_{ij}} = 2c \int_{-b}^{b} \int_{-\pi}^{\pi} [\, (H_x - H_{tx}) \frac{\partial H_x}{\partial a_{ij}} + (H_y - H_{ty}) \frac{\partial H_y}{\partial a_{ij}} + (H_z - H_{tz}) \frac{\partial H_z}{\partial a_{ij}}] \times d\phi dz = 0$ (2). The regularization technique [4] was used to solve the ill-conditioned matrix formed by equations such as Eq. (2).

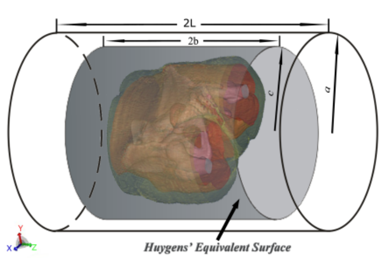

Fig.1 The cylindrical coordinate system including the source surface, the Huygens' equivalent surface, and the human pelvis electromagnetic model.

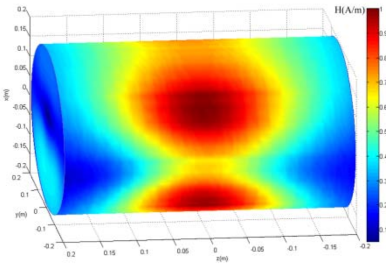

Fig.2 The normalized RF magnetic fields distribution on the Huygens' equivalent surface.

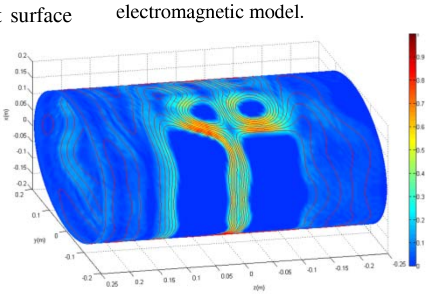

Fig.3 The obtained source current density distribution and the contour plotting of the current pattern.

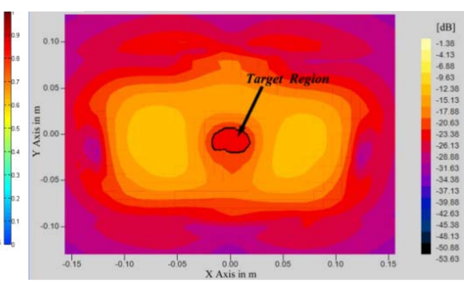

Fig.4, The induced RF magnetic field (B_1^+) inside the pelvic region by the obtained current source.

Results: The electromagnetic fields on the Huygens' equivalent surface subjected to the homogeneous target magnetic fields inside the pelvic region are shown in Fig. 2. The computational time to obtain the stabilized magnetic fields on the Huygens' equivalent surface was approximately six hours. Normalization to the maximum was applied to the obtained Huygens' equivalent surface, where the magnetic fields distribution was evidently inhomogeneous. The magnetic fields vector on the Huygens' equivalent surface had three orthogonal components (x, y, and z components) because of the complicated field-tissue interactions. The obtained source current density distribution and the contour plotting of the current pattern are shown in Fig. 3. The results in Fig. 3 showed that the current patterns were irregular, indicating the complicated current density distribution subjected to the inhomogeneous magnetic fields distribution on the Huygens' equivalent surface.

Discussion and Conclusion: To evaluate the obtained source current distribution, the induced RF magnetic field (B_1^+) inside the pelvic region was calculated using the FDTD solver [2], shown in Fig. 4. The maximum and minimum magnitude of magnetic fields inside the uterus were 0.657 and $0.595 A/m$, respectively. The homogeneity was 93%. The SAR was less than $4.65 W/kg$ over the entire pelvic region in any 1-gram tissue. The validation results showed that the obtained source current can definitely induce the desired goal field using the proposed hybrid method. The proposed hybrid inverse/FDTD computational electromagnetic method may be further implemented for flexible, goal-oriented, and optimized source RF field design for high-field MRI application. This demonstration for linear fields in a small target region avoids, to some degree, significant challenges relating to phase delays and attenuation cause by the human body. Future work will explore limitations for larger target regions and circularly-polarized fields, as well as methods to overcome them.

References: 1. P.T. While et al., *Meas Sci Technol*, 2005;16:1381-1393. **2**. Collins et al., *Magn Reson Med*, 2011;65:1470. **3.** P.T. While et al., *Meas Sci Technol*, 2007;18:245-259. **4** .B.G. Lawrence et al., *IEEE T Bio-Med Eng*, 2002;49:64-71.