Radiation Damping in a Detuned Probe by the Bloch-Kirchhoff Equations

James Tropp[1]
[1]Engineering, GE Healthcare Technologies, Fremont, CA, United States

The loss of signal amplitude, as energy is withdrawn from the spins and sent to the detector, is called radiation damping in NMR, and is fundamental to all NMR experiments. Without it there can be no NMR signal. Bloembergen and Pound (1) gave the original Bloch-Kirchhoff (B-K) equation, describing radiation damping, but, due to non-linearity, solved the equations only for steady state conditions. Quantitative applications were also hindered by parameterization in terms of the ill-defined filling factor. Later workers solved the B-K equations numerically (2), to describe chaotic behavior in the NMR laser; but application to the original problem of radiation damping has lagged.

It has long been known empirically that the rate of radiation damping is sharply reduced by detuning the probe, a famous example being given by Abragam (3); but we know of no prior theoretical treatment. The normal modified Bloch equations are adequate for describing radiation damping in a tuned probe, but introduction of detuning requires the full B-K approach. We here present and solve appropriate B-K equations for a detuned probe, using the reciprocity principle to eliminate the troublesome filling factor (4). The B-K equations are written below, for a tuned probe, using conventional symbols. The complex impedance resulting from detuning appears (after transformation) as an offset from the Larmor frequency (cf the 1, 2 and 2, 1 matrix elements below); and it must be given in terms of the in-phase and quadrature rotating frame components, without the use of the customary imaginary factor *i*.

$$\begin{bmatrix} \dot{I}_x \\ \dot{I}_y \\ \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{bmatrix} = \begin{bmatrix} -R/L & (\omega_0^2-\omega^2)/\omega & 0 & -\omega_0 V B_1(1)/2L & 0 \\ (\omega_0^2-\omega^2)/\omega & -R/L & \omega_0 V B_1(1)/2L & 0 & 0 \\ 0 & 0 & -1/T_2 & \Delta\omega & -\omega_{1y}(I_y) \\ 0 & 0 & -\Delta\omega & -1/T_2 & \omega_{1x}(I_x) \\ 0 & 0 & \omega_{1y}(I_y) & -\omega_{1x}(I_x) & 0 \end{bmatrix} \begin{bmatrix} I_x \\ I_y \\ M_x \\ M_y \\ M_z \end{bmatrix}$$

As an aside, the equations although highly non-linear, are numerically stable for perfect tuning of the probe, and may then be solved numerically in their given form. However, the offset term from detuning introduces instability, which we treat, following Brun et al. (2) by taking the steady state values of the currents at each time interval. Following our earlier work (4) with slight modifications, these equations are solved for protons in water at 14.1 T, with parameters corresponding to an inverse detection probe, L = 58 nH, source loaded R = 2 ohms, Q = 110 with source loading. The inverse T_2 is 10/sec.

Typical results are shown below for three values of the tuning offset: i) the probe tuned to the Larmor frequency (600 MHz), ii) the probe tuned to an offset frequency (607 MHz) giving 5 ohms reactance, and iii) the probe tuned to an offset frequency (614 MHz), giving 10 ohms reactance. The corresponding color sequence of the traces in the figures below is black, blue, red. Figure 1 gives the time course of the reception current in the probe (magnitude of $I_x + iI_y$) following a nutation of $\pi/2$; the horizontal axis is seconds, the vertical amperes. Note the significant drop with offset in the total current, as well as the slower decay, corresponding to a decrease in the rate of radiation damping. These results are borne out in the magnitude lineshapes (Fig. 2) which are scaled to the current, so that the peak integrals accurately depict the total energy delivered for the creation the reception current (Fig. 1). As expected, the lines narrow significantly as damping decreases.

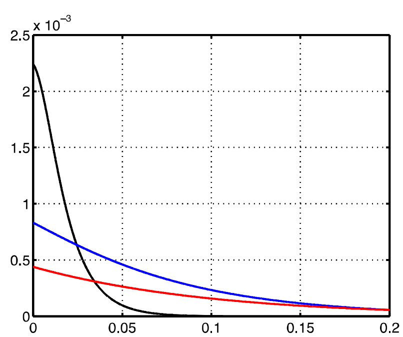

Fig. 1: Time course of total reception current.

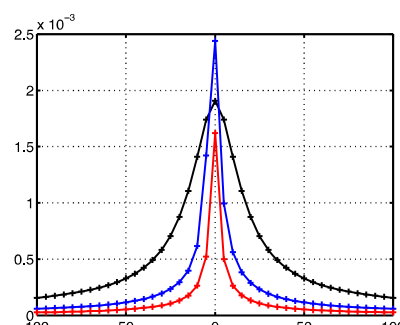

Fig. 2: Magnitude lineshapes for damping with detuning

References:

1. N. Bloembergen & R. Pound, Phys. Rev. 95, 8 (1954).
2. E. Brun et al., J. Opt. Soc. Am. B 2, 156 (1985).
3 A. Abragam, "The Principles of Nuclear Magnetism", Oxford (1961) p. 65.
4. J. Tropp & M. van Criekinge, J. Magn. Reson. 206, 161 (2010).