Direct Reconstruction of the Average Diffusion Propagator with Simultaneous Compressed-Sensing-Accelerated Diffusion Spectrum Imaging and Image Denoising by Means of Total Generalized Variation Regularization

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Purpose: Due to subsequent acquisition of multiple diffusion weightings and directions, a major challenge in diffusion MRI is balancing between acquisition duration, image resolution and signal-to-noise ratio (SNR). By mathematically incorporating assumptions about the images (so-called prior knowledge), regularized reconstruction techniques can be used to improve SNR, allow shorter acquisition durations [1,2], and improve image resolution [3]. State-of-the-art regularizations use the assumptions of similar image intensities for neighboring voxels in image space [4–7], or similar intensities for neighbors in diffusion-encoding q-space [2,6]. (Comparison of regularization effects in Ref. [8].)

Methods: In the present work, we regularize for neighborhood similarities both in image space and in q-space at the same time, while preserving egdes using a piecewise-smooth image model. This is a stronger formulation that allows better denoising than using either one of the assumptions (image space or q-space), or than applying both of them in subsequent independent steps. Furthermore, we incorporate q-space undersampling in a compressed sensing framework, allowing shorter acquisition durations. A third novelty is an optional modification of the encoding operator such that the average diffusion propagator (r-space) rather than the q-space is reconstructed (with regularization in r-space rather than q-space). This additionally links the diffusion-weighted images (DWIs) together not only by their structural similarity but also by the underlying Fourier relationship between q-space signal and the diffusion displacement probability density function. Reconstruction is performed directly from k-space data.

Our acquisition in q-space is Cartesian, i.e. diffusion spectrum imaging [9] (DSI), accelerated by random undersampling and thus amenable to compressed-sensing reconstruction [2]. The regularization used is 2$^{\text{nd}}$-order total generalized variation [10,4] (TGV), allowing locally affine (i.e. smooth) image regions as well as edges, while removing random noise. We propose applying TGV to all five data dimensions (2-D image space and 3-D diffusion space).

Image reconstruction with TGV regularization [10,4] is performed by solving

$$\arg\min_{\boldsymbol{\rho}} \|\mathbf{E}\boldsymbol{\rho} - \boldsymbol{d}\|_2^2 + \mathrm{TGV}_\alpha^2(\boldsymbol{\rho}),$$

where we choose $\mathbf{E}=\mathcal{F}_{x\to k}\mathbf{C}\mathbf{b}e^{i\boldsymbol{\varphi}}\mathcal{F}_{r\to q}$ to be the encoding operator to average propagator space (with $\mathcal{F}_{r\to q}$ Fourier transform from r-space to undersampled q-space, $\boldsymbol{\varphi}$ complex image phase estimated from standard reconstruction (to correct for phase disturbances in the theoretically phase-free q-space), $\mathbf{b}$ multiplicative intensity inhomogeneity field estimated using a body-coil b=0 image, $\mathbf{C}$ coil sensitivity weightings, and $\mathcal{F}_{x\to k}$ Fourier transform from image space (x-space) to k-space), $\boldsymbol{\rho}$ is the reconstructed data in (five-dimensional) x-r-space, $\boldsymbol{d}$ is the acquired data in k-q-space, and

$$\mathrm{TGV}_\alpha^2(\boldsymbol{\rho}) = \min_v \alpha_1 \int_\Omega |\nabla\boldsymbol{\rho} - v|\,\mathrm{d}x + \alpha_0 \int_\Omega |\mathcal{E}(v)|\,\mathrm{d}x$$

is the 2$^{\text{nd}}$-order TGV regularization balancing the first and second derivative of the image via the complex vector field v (with $\mathcal{E}(v) = (\nabla v + \nabla v^{\mathrm{T}})/2$ the symmetrized derivative and Ω the field of view in x-r-space). The discretized solution is obtained with a first-order primal-dual algorithm [11]. For q-space reconstruction, $\mathcal{F}_{r\to q}$ is omitted.

For experiments, healthy volunteer DSI data with a single coil and a 32-channel head coil were acquired on a 3T GE MR750 clinical MR scanner (GE Healthcare, Milwaukee, WI, USA). Undersampled 11^3-DSI with acceleration R=3.4, $b_{\max}$=2000mm/s²: single Tx/Rx head coil (TE=96ms, TR=3s, 128x128, FOV=24cm, slice=4mm). Full 11^3-DSI at $b_{\max}$=8000mm/s²: 32-channel head coil (single spin echo, TE=124.3ms, TR=1.6s, 96×96, FOV=24cm, slice=2.5mm).

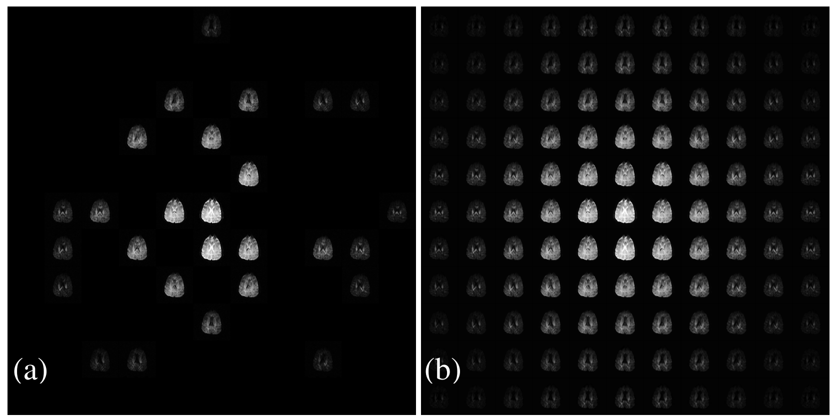

Figure 1: Four-dimensional slice of reconstructed five-dimensional data (image space and q-space; single coil, $b_{\max}$=2000mm/s²). **(a)** Standard image reconstruction of DSI data, random undersampling [2] with acceleration factor R=3.4. **(b)** Direct average-propagator reconstruction of the same randomly undersampled DSI data, going back to q-space in the last step (TGV regularization parameters: $\frac{1}{2}\alpha_0=\alpha_1=0.0008$).

Results: The reconstructed x-r-space data, transformed to x-q-space for better comparison with standard DWI-wise reconstruction, are shown in Figs. 1–3. Fig. 1 demonstrates the capability of direct average-propagator reconstruction to recover data with q-space undersampling. An acceleration factor of R=3.4 is feasible and yields meaningful recovery of missing q-space data. Fig. 2 demonstrates the denoising effect (achieved simultaneously with the undersampling reconstruction of Fig. 1). Fig. 3 shows the performance at high b-value (low SNR) for different regularization levels. Features that are difficult to see in noisy images are well recovered by using image space and q-space information.

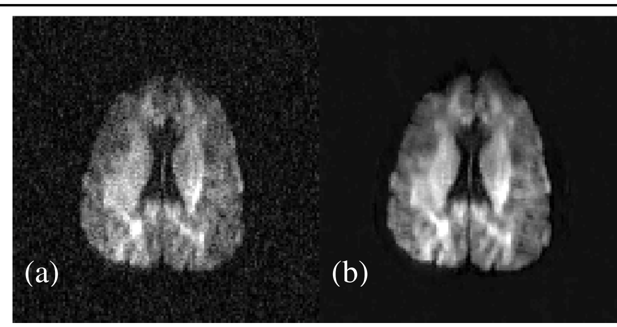

Figure 2: In addition to the q-space undersampling reconstruction shown in Fig. 1, each DWI (shown here: q=(-5;0;0)) is also denoised during direct reconstruction of undersampled DSI data, yielding improved SNR. **(a)** Standard reconstruction. **(b)** Direct reconstruction.

Discussion and Conclusion: Our experiments have demonstrated the feasibility of direct reconstruction of the average diffusion propagator with five-dimensional TGV regularization, along with the method's ability of undersampled q-space compressed sensing reconstruction at acceleration factor R=3.4, and DWI denoising. Future work may focus on studying the influence of diffusion weighting $b_{\max}$ (which influences the estimated propagator), acceleration factor R and regularization parameters α_1 and α_0 on the reconstruction and on quantitative parameters using different diffusion models.

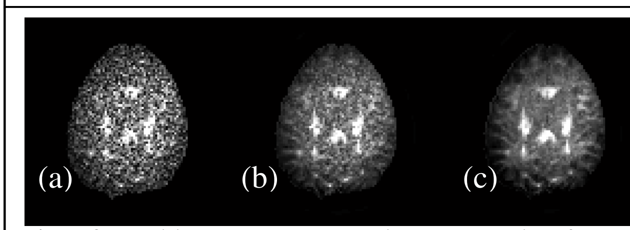

Figure 3: Denoising by 6-D TGV-regularized reconstruction of r-space of fully-sampled DSI data at $b_{\max}$=8000mm/s² (shown here: q=(0;5;0) in reciprocal q-space). **(a)** Standard reconstruction. **(b)** Direct reconstruction with $\frac{1}{2}\alpha_0=\alpha_1=0.002$. **(c)** Direct reconstr. with $\frac{1}{2}\alpha_0=\alpha_1=0.004$.

References: [1] Lustig *et al.*, MRM 2007. [2] Menzel *et al.*, MRM 2011. [3] Scherrer *et al.*, MIA 2012. [4] Knoll *et al.*, MRM 2011. [5] Haldar *et al.*, MRM 2012. [6] Sperl et al., ESMRMB 2012. [7] Golkov *et al.*, ESMRMB 2013, #153. [8] Golkov *et al.*, ESMRMB 2013, #23. [9] Wedeen *et al.*, MRM 2005. [10] Bredies *et al.*, SIAM J Imaging Sci 2010. [11] Pock, Cremers *et al.*, ICCV 2009. **Grant support:** Deutsche Telekom Stiftung.