Parallel Reconstruction using Patch based K-space Dictionary Learning

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Introduction: Recently, a parallel reconstruction technique SAKE[1] has been developed using Singular Value Decomposition (SVD) to impose low rank property without using auto-calibrating signal (ACS) data, which can further improve the result provided by SPIRiT[2] with sufficient sampled ACS area. Although predetermined SVD factor analysis method has good analytical and numerical properties, it has been demonstrated[3] that a learned dictionary can better adapt to acquired data and improve the reconstruction result. In this study, we propose a new patch-based dictionary learning method to estimate the local signal features in k-space and demonstrate its improved performance in-vivo.

Theory: Low rank matrix completion can be solved by singular-value thresholding[1]. Assuming that $A = U\Sigma V^H \approx U_t\Sigma_t V_t^H = U_t W$, the missing entries of a low rank matrix A can be completed using a linear combination of the basis column vectors in U_t, where U_t corresponds to the left singular matrix U after hard thresholding. Also, the weighting coefficient matrix W would be sparse due to the low rank property of Σ_t. This equivalently entails presence of a dictionary that can be used to describe this low rank matrix with sparse coefficients. Similar to SAKE, we divided k-space into N_p overlapping patches and define y_i as a vector of the acquired k-space data with patch index i from 1 to N_p, P_i as the under-sampling pattern for ith patch, D as a dictionary matrix with $\|d_k\|_2^2 \le 1, k = 1,2,\cdots,K$ constraints applied on its column vectors, and w_i as a sparse vector for ith patch. The KDL method can be formulated as:

$$\min_{D\in\mathscr{D},W} \sum_{i=1}^{N_p} \|y_i - P_i D w_i\|_2^2 + \mu \|w_i\|_1, \mathscr{D} = \left\{ D \mid \|d_k\|_2^2 \le 1, k = 1,2,\cdots,K \right\}$$

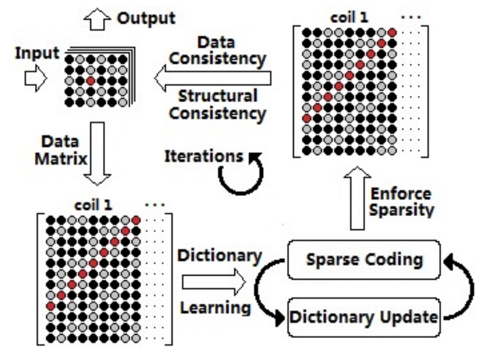

Figure 1. Flowchart of K-space Dictionary Learning.

The Projection Onto Convex Sets (POCS) method with block proximal-gradient dictionary learning[4] can be used to solve the above problem as shown in Figure 1.

Methods: The brain data of a healthy volunteer was scanned on a 3T whole body MR system (Philips Achieva, R3.2.1, Netherland) with standard eight-channel head coil. 3D MPRAGE sequence was performed with the following parameters: FOV = 200x200x200mm^3, voxel size = 1x1x1mm^3, TR/TE = 10/4.1ms, IRTR/TI = 998/498ms, FA = 11, TFE factor = 98, linear encoding, no fat suppression. The fully sampled k-space data was retrospectively x3/x4 undersampled with Poisson Disk Sampling (PDS) pattern with calibration area $4\times4/26\times26$. The reconstruction result of the proposed KDL method was compared with both SPIRiT and SAKE using the normalized root mean square error (nRMSE). All three methods used the same 7×7 kernel/patch size. In SAKE, the singular-value threshold was fixed to 60. In KDL, the dictionary size varied from 60 to 256 was used to test the impact of dictionary scale and the initial dictionary was trained by SVD. Other relevant parameters were optimized to achieve best performance for each reconstruction method.

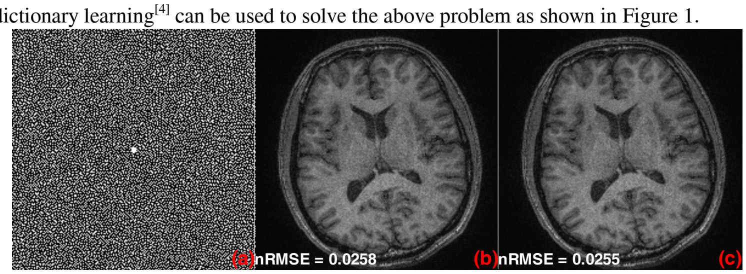

Figure 2. Performance Comparison at x3 Acceleration with insufficient ACS data. (a) PDS sampling mask (b) SAKE (c) KDL with dictionary size 60

Results and Discussion: In x3 accelerated calibrationless scenario, the KDL method outperforms the reconstruction result of SAKE with smaller nRMSE (Figure 2). In the case of data with sufficient ACS data, the KDL method also provides less error than the other two methods with x4 acceleration factor (Figure 3). Notably, the estimation error of KDL can be further reduced with a larger dictionary size since more complex/detail information will be represented but at the cost of more total iteration numbers.

Conclusion: In this work, we present a new KDL method workable for both calibrationless and autocalibrating parallel reconstruction. We demonstrated that the adaptive sparse transform/learned dictionary in k-space can further improve the image quality for parallel reconstruction.

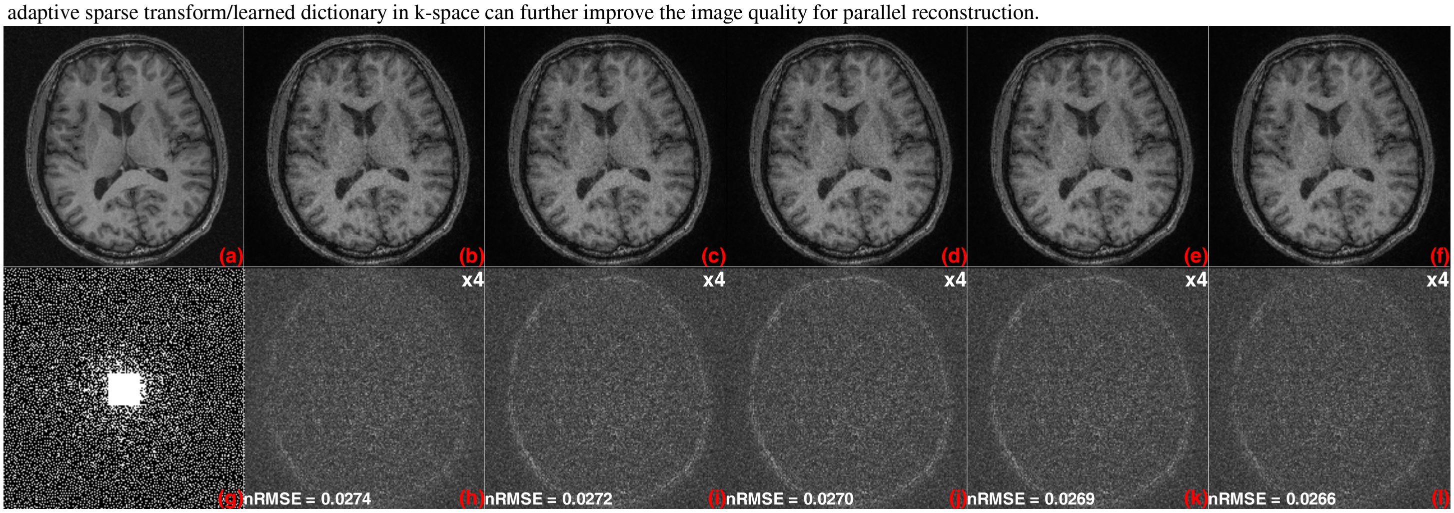

Figure 3. Performance Comparison at x4 Acceleration with ACS data. (a) full (b) SPIRiT (c) SAKE (d)~(f) KDL with dictionary size 60, 128 and 256. (g) PDS sampling mask (h)~(l) reconstruction error between (a) and its above row with 4 fold magnification.

References: [1] P. Shin, et al. Calibrationless parallel imaging reconstruction based on structured low-rank matrix completion, 2013, accepted to Magn Reson Med. [2] M. Lustig, et al. Magn Reson Med. 2010 Aug;64(2):457-71. [3] S. Ravishankar, et al. IEEE Trans Med Imaging. 2011 May;30(5):1028-41. [4] Y. Xu, et al. A fast patch-dictionary method for whole-image recovery, UCLA CAM report 13-38, 2013.