

# Optimal auto-calibration kernel estimation using double adaptive weights

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**Target Audience:** MR researchers and clinical scientists working on Parallel Imaging

**Purpose:** The estimation of GRAPPA and SPIRiT auto-calibration kernel, which is usually formed as an inverse problem, is an essential step for Parallel Imaging (PI)<sup>1-2</sup>. Regularizations<sup>2-4</sup> for the kernel coefficients have been discussed before to achieve more accurate kernel estimation. However, the weighting for each measurement in the inverse problem has not been fully discussed. In this work, we propose a novel scheme for auto-calibration PI, which consider both measurement and kernel coefficients to achieve an optimal solution under a statistical model. Experiments compared with previous proposed solution and demonstrate advantages in kernel value constraints and reconstruction accuracy.

**Theory:** The computation of GRAPPA kernel<sup>1</sup>  $\mathbf{g}$ , or equivalent auto-calibration kernel in other algorithm (SPIRiT<sup>5</sup>, etc.) is commonly formed as an inverse problem: estimate linear interpolation kernel coefficients  $\mathbf{G}$  from measured  $k$ -space samples  $\mathbf{X}$  and  $\mathbf{y}$  given  $\mathbf{X}\mathbf{g} = \mathbf{y}$ . To solve the  $\mathbf{g}$ , a set of auto-calibration data is fully sampled in the center  $k$ -space to use as measurements and form  $\mathbf{X}$  and  $\mathbf{y}$  by reshaping the matrix properly. In order to prevent noise amplification due to the high condition number, Least-square with Tikhonov regularization was usually used. Advanced regularizations<sup>3,4</sup> for the kernel coefficients were also proposed, formulated as  $\min_{\mathbf{g}} \|\mathbf{X}\mathbf{g} - \mathbf{y}\|_2 + \|\mathbf{W}\mathbf{g}\|_{1,2}$ , in which  $\mathbf{W}$  is adaptive weights for kernel coefficients. However, since  $\mathbf{X}$  is formed by reshaping moving patches, each patch is not independent from others when they are closed. Besides, the linear interpolation is not optimal since the noises are also correlated. To tackle this problem, we explored a statistical model and used two adaptive weights both for the measurement and kernel coefficient to achieve the Maximum Likelihood estimation.

**Method:** We modeled the linear interpolation with noises ( $n_y, n_x$ ), which also includes fitting errors,  $\mathbf{y}_{\text{real}} = \mathbf{X}_{\text{real}}\mathbf{g} + \mathbf{n} \rightarrow \mathbf{y} - \mathbf{n}_y = (\mathbf{X} - \mathbf{n}_x)\mathbf{g}$ . The Maximum Likelihood estimation was:

1) fix  $\mathbf{g}$ : noiseless  $\mathbf{y}$  is  $\hat{\mathbf{y}}_{\text{real}} = \mathbf{E}(\mathbf{y} - \mathbf{n}_y | \mathbf{X}; \mathbf{g}) = \mathbf{X}\mathbf{g}$ .  
 2) fix  $\mathbf{y}$ :  $\hat{\mathbf{g}} = \mathbf{argmin}(\mathbf{y} - \hat{\mathbf{y}}_{\text{real}})^T \Sigma_n^{-1} (\mathbf{y} - \hat{\mathbf{y}}_{\text{real}}) + (\mathbf{W}\mathbf{g})^T (\mathbf{W}\mathbf{g})$ ,  
 In which,  $\mathbf{W}$  is diagonal adaptive weights matrix based on  $g_i \sim \mathcal{N}(0, w_{i,i}^{-1})$  and  $\Sigma_n$  is the Covariance matrix of  $(\mathbf{n}_x\mathbf{g} + \mathbf{n}_y)$ . Clearly, there are two different noise related weights:  $\Sigma_n^{-1}$  for the measurement and  $\mathbf{W}$  for the coefficients. The analytical solution is:  $\hat{\mathbf{g}} = (\mathbf{X}^T \Sigma_n^{-1} \mathbf{X} + \mathbf{W}^T \mathbf{W})^{-1} \mathbf{X}^T \Sigma_n^{-1} \mathbf{y}$ . The optimal solution can be achieved using a few iterations since  $\Sigma_n^{-1}$  and  $\mathbf{W}$  can be better estimated from more accurate  $\mathbf{g}$ . The Covariance matrix was updated efficiently due to its sparse structure and the adaptive weights  $\mathbf{W}$  was initialized using (3) and updated based on (4).

**Results:** An axial brain scan with an eight channel head coil was used to validate the calibration scheme for auto-calibration PI. The datasets were all fully sampled and retrospectively under-sampled along PE direction. To evaluate noise effect, artificial noise was added in the channel-data based on selected SNR. Tikhonov regularization and Adaptive regularization were implemented as comparison. One estimated Covariance Matrix  $\Sigma_n$  was shown in figure 1. The distribution of kernel coefficients was demonstrated in figure 2 and the reconstruction error using different calibration with the same reconstruction algorithm (GRAPPA and SPIRiT) was compared in figure 3.

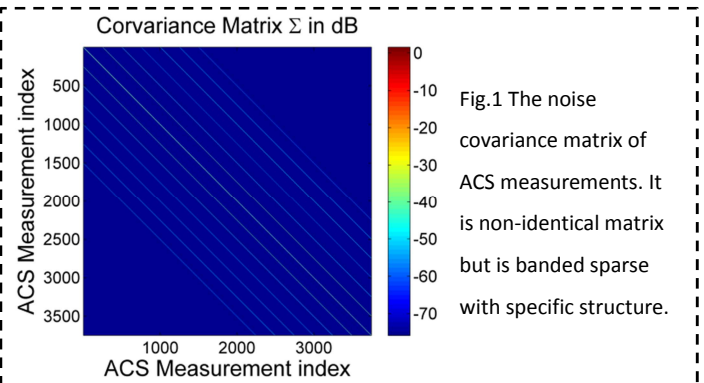


Fig.1 The noise covariance matrix of ACS measurements. It is non-identical matrix but is banded sparse with specific structure.

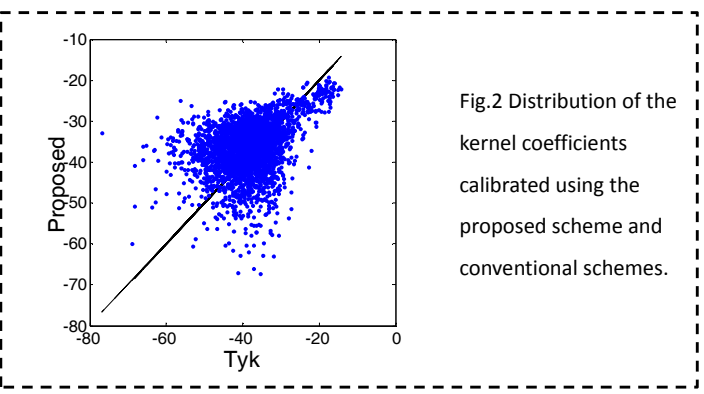


Fig.2 Distribution of the kernel coefficients calibrated using the proposed scheme and conventional schemes.

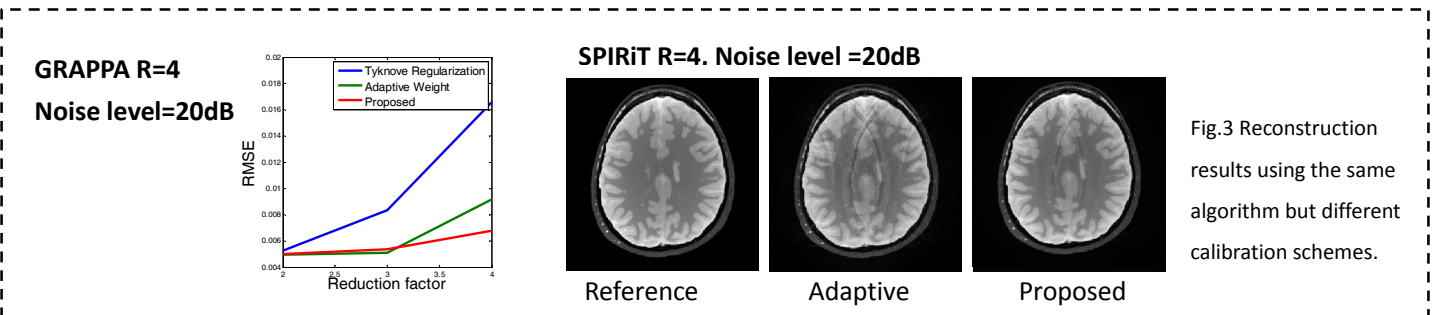


Fig.3 Reconstruction results using the same algorithm but different calibration schemes.

**Discussion:** This work models the calibration as an inverse problem of linear interpolation with considerable noise. Two weights both for measurement and kernel coefficients are explicitly derived to achieve statistical optimal solution for the inverse problem. In application, double adaptive weights were used to regularize the Least-square and iterative scheme was applied to reach the convergence of optimal solution. In-vivo experiment data demonstrate the advantage of the proposed scheme for decreased coefficient error, reconstruction error and noise level. Further investigation to improve and valid proposed scheme is undergoing.

**Conclusion:** We presented a novel scheme to estimate auto-calibration kernel coefficients by using iterative double adaptive weights for both ACS measurements and kernel coefficient. Results demonstrate advantages over existing calibration scheme.

**References:** [1] Griswold, et.al. MRM 2002 [2] Lin et.al. MRM 2011 [3] Liu et.al. MRM 2013 [4] Jin et.al ISMRM 2013 [5] Lustig et. al. MRM 2010