Discrete shearlets as a sparsifying transform in a split Bregman reconstruction of low-rank plus sparse component from undersampled (k, t)-space small bowel data

Nikolaos Dikaios[1], Benjamin Tremoulheac[1], Alex Menys[2], Simon Arridge[1], and David Atkinson[2]

[1]*Dept. of Medical Physics and Bioengineering, University College London, London, Great London, United Kingdom,* [2]*Centre of Medical Imaging, University College London, Great London, United Kingdom*

Aim: Quantification of small bowel motility correlates with disorders such as Crohn's disease [1]. The motility metric can be derived from non rigid registration of dynamic MR images, and its accuracy depends on their spatial/temporal resolution. We propose a split Bregman algorithm [2] to reconstruct alias free dynamic MR images from undersampled (k,t)-space data, improving either the temporal resolution or maintaining the same temporal resolution and improving the spatial resolution. The proposed algorithm uses shearlets as an optimal sparsifying transform [3], and assumes that the recovered image consists of a low-rank plus a sparse component. A potential advantage of the proposed algorithm is that it could separate the respiratory motion (low rank) from the bowel motility (sparse), allowing us to calculate motility metrics solely dependent on the bowel motility.

Theory: Discontinuities/edges in multidimensional data need many wavelet coefficients to be accurately represented; hence wavelet representations are not sparse. Shearlets [4,5] use the framework of affine systems and are a non-isotropic version of the wavelet transformations that can provide an optimal sparse representation of images. Shearlets $\psi \in L^2(R^2)$ are directional representation systems [4,5] generated by $\{\psi_{\alpha,s,\tau}(r) = |detD|^{-1/2}\psi(D^{-1}(r-\tau))\}$, where $\alpha > 0$ is the scaling parameter, $s \in R$ is a shear parameter, $\tau \in R^2$ is the translation parameter and $D = [\alpha \quad -s\sqrt{\alpha}; 0 \quad \sqrt{\alpha}]$ (details in [5]). The shearlet transform of any given image $x \in L^2(R^2)$ can be defined as $Sh_{\alpha,s,\tau}(x) = \langle x, \psi_{\alpha,s,\tau} \rangle$. Discrete shearlets on the cone treat all directions uniformly and were preferred in this framework.

Reconstruction: In this work shearlets were employed as a sparsifying transform in a split Bregman reconstruction [2] of undersampled (k,t)-space small bowel data. The optimization problem is formulated as

$$min_{L,S} \frac{1}{2} \|F_u(L+S) - y\|_2 + \mu(\|L\|_* + \lambda \|Sh_{\alpha,s,t}(S)\|_1)$$

where $\|\cdot\|_1$ is the $\ell 1$-norm, $\|\cdot\|_*$ is the nuclear norm, $x=L+S$ is the recovered image, F_u is the undersampled Fourier transformation, y is the measured k-space. The aim is to recover low-rank L and sparse component S of the image. A shearlet transformation $Sh_{\alpha,s,t}$ (in space) was included for the sparse component S. The parameter λ is a trade-off parameter between the low-rank L and the sparse component S [6], for this problem we set $\lambda = 1/\sqrt{\max(n,m)}$ to recover a low-rank incoherent matrix, where n is the number of spatial pixels, and m is the number of dynamic acquisitions. Similarly μ is a trade-off parameter between the data consistency (first) term and the decomposition (second) term, and was set to μ=10. The proposed algorithm is denoted as ktSBR, and was compared against focal underdetermined system solver (ktFOCUSS) [7]. The parameter settings for ktFOCUSS were 40 inner iterations, 2 outer iterations, weighting matrix power factor 0.5, and a zero-filled fast Fourier transformation (zf-FFT) of the undersampled data as an initial estimate.

MR protocol and (k,t)-data generation: A Balanced Turbo Field Echo (BTFE) sequence was used to acquire a 15cm coronal volume through the abdomen and pelvis with a torso coil for a volunteer (2.5x2.5x6mm resolution, FA 20, TE=1.7ms, TR=3.6ms, SENSE (3LR, 1.5AP), no slice gap). The volunteer was scanned using a Philips Achieva 3T MRI scanner while breath-hold followed by free-breathing (120 images were acquired with temporal resolution 1 second). The original scanner reconstructed images were projected to (k, t)-space with FFT where normally distributed noise was added. For the retrospective undersampling pattern, phase encoding lines were randomly selected per volume and per time frame. The centre of k-space was more densely sampled. Undersampling patterns for 8-fold acceleration were generated using a Monte-Carlo algorithm to generate a sampling pattern with minimum peak interference [8].

Results: Figures 1 and 2 evaluate the ktSBR versus the zf-FFT and ktFOCUSS. The mean absolute percentage difference between the ground truth dynamic MR and the reconstructed images using ktFOCUSS/ktSBR from 8-fold undersampled data is 18.4/16.8% for the breath-hold period and 19.1/17.3% for the free-breathing period.

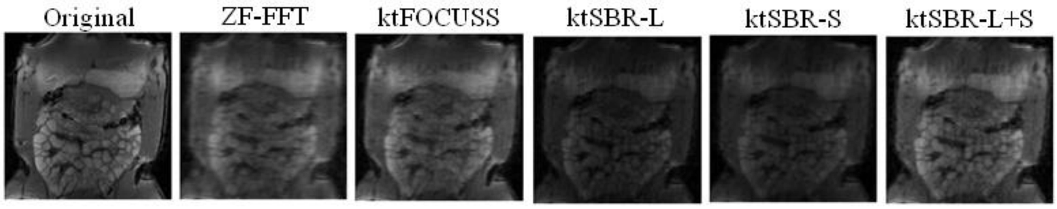

Fig. 1. Reconstructed images from undersampled (k-t) space data during free-brathing using FFT, ktFOCUSS, ktSBR for 8-fold undersampling factors.

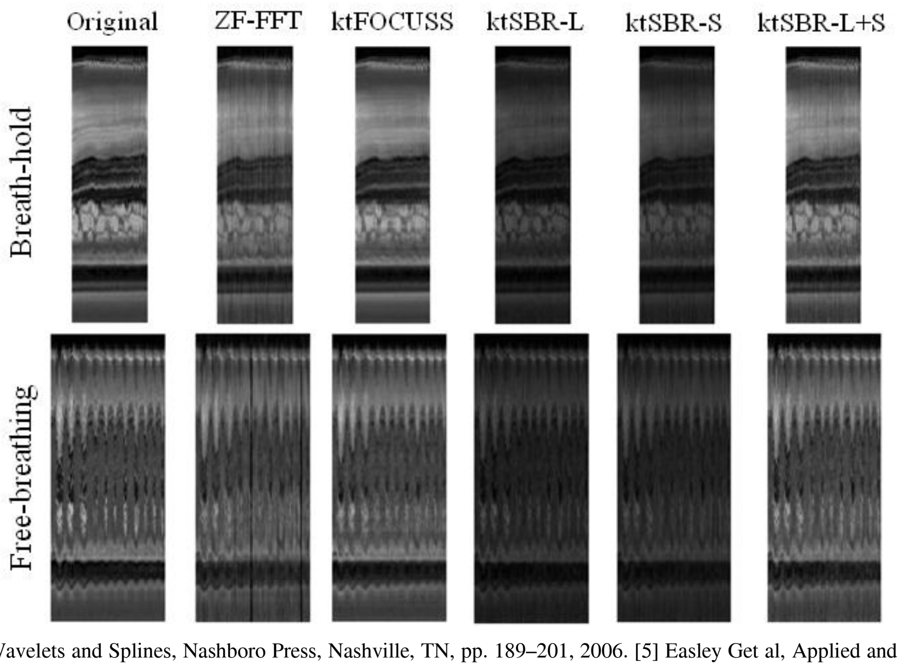

Fig. 2. Time-cut representation (y-t space) of reconstructed images with FFT, ktFOCUSS and ktSBR for 8-fold undersampling (acceleration) factors.

Conclusions: Results indicate that both ktSBR and ktFOCUSS significantly reduce the aliasing artifacts due to undersampling. Quantitative comparison between the ground truth dynamic MR images and the reconstructed ones indicate that ktSBR slightly outperforms ktFOCUSS, but ktSBR is more computationally expensive.

The expectation of separating respiration (low rank) from bowel motility (sparse component) was not clearly observed in ktSBR (Fig 2).

Future work will involve characterization of the regularization parameters μ, λ using the L-curve criterion [9], and combination of ktSBR with other undersampling techniques like parallel imaging [10] and halfscan.

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