Improved Blind Compressed Sensing for Dynamic MRI Reconstruction

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INTRODUCTION

Recently the Blind Compressed Sensing (BCS) [1] framework was used for offline dynamic MRI reconstruction [2]. It models the partial K-space data acquisition for frames t=1..T as: $vec(Y) = \Phi\ vec(X) + \eta$, where $Y = [y_1 | ... | y_T]$, $X = [x_1 | ... | x_T]$, $\Phi = BlockDiag(F_t)$, t=1...T ,F_t being a partial Fourier operator and $\eta \sim N(0,\sigma^2)$ is the system noise. The BCS models X as: $X = DS$, where both D, the dictionary and S, the sparse coefficients, are unknowns and needs to be estimated.

Owing to temporal correlation between the subsequent frames, the matrix X is low-rank [3], therefore DS should be low-rank as well. However, the BCS reconstruction scheme does not account for the rank deficiency of X or that of D and S. In this work, we model the rank-deficiency of X by imposing low-rank constraints on the coefficient matrix S.

THEORY

BCS reconstruction is formulated as: $\min_{D,S} \|vec(Y) - \Phi\ vec(DS)\|_2^2 + \lambda_1 \|D\|_F^2 + \lambda_2 \|S\|_1$. This work proposes to improve upon it by imposing a low-rank (nuclear norm) penalty on S. Thus, we propose to solve: $\min_{D,S} \|vec(Y) - \Phi\ vec(DS)\|_2^2 + \lambda_1 \|D\|_F^2 + \lambda_2 \|S\|_1 + \|S\|_*$.

We solve this by Bregman type variable splitting with Alternating Directions Method of Multipliers (ADMM) [4]. We introduce three proxy variables - P, Q and R for the three penalty functions respectively. We add terms relaxing the equality constraints of each quantity and its proxy, and in order to enforce equality at convergence, we introduce Bregman variables B_1, B_2 and B_3. The new objective function is: $\min_{D,S,P,Q,R} \|vec(Y) - \Phi\ vec(DS)\|_2^2 + \lambda_1 \|P\|_F^2 + \lambda_2 \|Q\|_1 + \lambda_3 \|R\|_* + \gamma_1 \|P - D - B_1\|_F^2 + \gamma_2 \|Q - S - B_2\|_F^2 + \gamma_3 \|R - S - B_3\|_F^2$

This allows the problem to be split into an alternating minimization of the following (easier) subproblems:

(1) $\min_D \|vec(Y) - \Phi\ vec(DS)\|_2^2 + \gamma_1 \|P - D - B_1\|_F^2$; (2) $\min_S \|vec(Y) - \Phi\ vec(DS)\|_2^2 + \gamma_2 \|Q - S - B_2\|_F^2 + \gamma_3 \|R - S - B_3\|_F^2$;

(3) $\min_P \lambda_1 \|P\|_F^2 + \gamma_1 \|P - D - B_1\|_2^2$; (4) $\min_Q \lambda_2 \|Q\|_1 + \gamma_2 \|Q - S - B_2\|_F^2$; (5) $\min_R \lambda_3 \|R\|_* + \gamma_3 \|R - S - B_3\|_F^2$.

Problems (1) to (3) are simple least squares problems with closed form solutions; but in practice will be solved by Conjugate Gradient. The problem (4) is an l_1-norm minimization problem and solving it is straightforward via shrinkage (soft-thresholding) [4], i.e. $Q_i = signum(S_i + B_i)\max(0, |S_i + B_i| - \frac{\lambda_2}{2\gamma_2})$. The solution to (5) is also straightforward (albeit less well known) via singular value shrinkage. $\Sigma_{R_i} = signum(\Sigma_{S_i} + \Sigma_{B3_i})\max(0, |\Sigma_{S_i} + \Sigma_{B3_i}| - \frac{\lambda_3}{2\gamma_3})$ where $R = U_R \Sigma_R V_R^T$, $S = U_S \Sigma_S V_S^T$ and $B_3 = U_{B3} \Sigma_{B3} V_{B3}^T$.

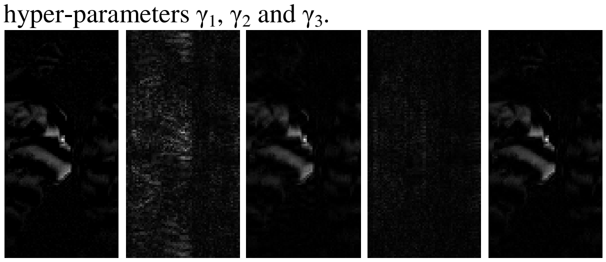
The final step is to update the relaxation variables: $B_1 \leftarrow P - D - B_1$; $B_2 \leftarrow Q - S - B_2$; $B_3 \leftarrow R - S - B_3$.

METHOD

The DCE dataset was acquired from a mouse bearing HCT-116 tumour. All images were acquired on a 7T/30 cm bore MRI scanner (Bruker, Germany). FLASH was used to acquire fully sampled 2D DCE-MRI data from the implanted tumour with 42.624 × 19.000 mm field of view, 128 × 64 matrix size TR/TE = 35/2.75 ms, 40° flip angle. 1200 repetitions were performed at 2.24 s per repetition.

RESULTS

In this study, our proposed methods were compared against the recently proposed BCS technique [2]. For both the experiments, variable density random sampling with an acceleration factor of 4 was simulated for partial sampling of the K-space. Of the 1200 frames, the first 200 frames were used for tuning the parameters λ_1, λ_2 and λ_3. Our algorithm is not very sensitive to the choice of hyper-parameters γ_1, γ_2 and γ_3.



Reconstructed 505th Frame. From L to R: ground truth (fully sampled K-space); difference image (ground truth and reconstructed) from BCS; reconstructed image via BCS; difference image from proposed method; reconstructed image from proposed method.

It is easy to observe from the difference images that BCS produces considerably more reconstruction artifacts compared to our proposed technique. In terms of quantitative reconstruction accuracy, the Normalized Mean Squared Error for the test dataset (frames 201 to 1200), from BCS is **0.156** and from our proposed method is **0.082**- this is a considerable improvement.

CONCLUSIONS

In this work, we have improved upon the recently proposed dynamic MRI reconstruction techniques based on BCS by introducing low-rank constraints on the coefficient matrix. We solved the optimization problem via variable splitting using ADMM.

REFERENCES

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