Optimized *k-t* Sampling for Combined Parallel Imaging and Compressed Sensing Reconstruction

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Introduction: The combination of parallel imaging (PI) [1,2] and compressed sensing (CS) [3] has shown improved reconstruction performance [4,5] as compared to applying either of the two methods alone. In many CS approaches, sampling patterns are designed to fully sample the center of k-space while random undersampling with decreasing density at higher phase encodes is used. Optimization of k-space trajectories combining regular and random undersampling patterns has been shown to be beneficial to improving reconstruction accuracy [6,7]. In dynamic imaging, time-interleaved *k-t* sampling may be used in addition to reduce signal overlaps in the spatial temporal Fourier domain (*x-f*) [8]. Following ideas presented in [6] and [8], we propose a Cartesian *k-t* sampling scheme for dynamic MRI with time-interleaved regularly and randomly undersampled low- and high-frequency components, respectively. Using cardiac short-axis data, it is demonstrated that this approach improves image reconstruction relative to standard density-weighted CS trajectories.

Theory: *L1*-norm regularized SENSE-based reconstruction estimates a coil-combined image $\boldsymbol{\rho}$ by minimizing the following optimization problem: $\|\mathbf{d}-\mathbf{E}\boldsymbol{\rho}\|_2^2+\lambda\|\boldsymbol{\Psi}\boldsymbol{\rho}\|_1$ (1) with the data vector $\mathbf{d}$, encoding matrix $\mathbf{E}$, sparsifying transform $\boldsymbol{\Psi}$ and regularization parameter λ. For dynamic 2D Cartesian imaging $\mathbf{E}=(\mathbf{I}_{Nc}\otimes\mathbf{I}_u\mathbf{F}_\mathbf{x})\mathbf{S}$ is composed of coil sensitivity weighting $\mathbf{S}$, Fourier transformation (FT) $\mathbf{F}_\mathbf{x}$ along the spatial coordinates $\mathbf{x}=(x,y)$, and undersampling of each coil's (N_c: Number of coils) *k-t* space using the reduced identity matrix $\mathbf{I}_u$. The point-spread function (PSF) of a conventional variable density weighted CS *k-t* trajectory [5] is shown in Fig.1 with noise-like aliasing along the phase encoding and, in *x-f* space, additionally along the temporal frequency direction (providing sparse domain incoherence).

With the proposed method, the fully sampled center of *k-t* space is replaced with a regular time-interleaved pattern while the degree of incoherent random undersampling is increased for higher phase encodes (Fig.1). The corresponding PSF in the spatio-temporal domain features well-separated peaks generating coherent aliasing, which is well suited for PI. Upon temporal FT $\mathbf{F}_t$, the coherent Nyquist replicas are shifted along the temporal frequency direction [8]. Thereby, the *x-f* PSF replicates the spectral support along the diagonal reducing coherent signal overlap while preserving incoherent interference.

Methods: Breath-held fully sampled cine 2D balanced SSFP short axis view data were acquired (28 coils, FOV: 296x264x8mm^3, spatial resolution: 2x2x8 mm^3) in healthy subjects on a 3T scanner (Philips Ingenia, Philips Healthcare, Best, The Netherlands). Data were compressed to 12 virtual channels [9]. Coil sensitivities were computed from a temporal averaged image using ESPIRiT [10]. *k-t* space was retrospectively decimated (5- and 8-fold) using time-interleaved regular 2-fold and increasing random variable-density undersampling of low and high spatial frequencies, respectively. Standard random variable density sampling with a full center of k-space [3,5] was used for comparison. Image series $\boldsymbol{\rho}$ were reconstructed with $\boldsymbol{\Psi}=\mathbf{F}_t$ using an iterative soft-thresholding algorithm [11] leaving the acquired data unchanged. Normalized root mean square errors (NRMSE) were calculated according to: $\|\mathbf{m}_r-\mathbf{m}_o\|_2/\|\mathbf{m}_o\|_2$ with vectors $\mathbf{m}_r$ and $\mathbf{m}_o$ stacking reconstructed and reference magnitude pixels, respectively.

Results: Fig. 2 illustrates temporal profile plots across the heart of the fully sampled reference and reconstructions from undersampled data. Fig. 3 shows systolic and diastolic magnitude frames together with the corresponding error maps.

Discussion: An optimized *k-t* sampling pattern comprising of regular time-interleaved and random undersampling has been proposed for combined PI and CS reconstruction. Improved unfolding and temporal resolution relative to standard CS sampling has been demonstrated. Even though sensitivity information in this study was not obtained directly from decimated data, temporally resolved autocalibration data can be easily derived by averaging of adjacent frames or via low-rank matrix completion [12]. The application to k-space based methods (SPIRiT) [4], 3D imaging or further sparsification of the *x-f* representation is straightforward.

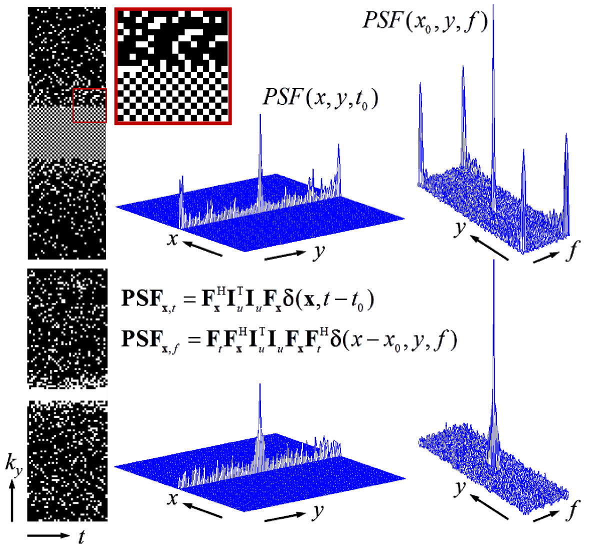

Figure 1 Illustration (neglecting sensitivity weighting) of a single coil's PSF in the *x-t* and *x-f* domain showing the undersampling artifacts of the proposed (top) and the conventional (bottom) *k-t* sampling scheme. $\boldsymbol{\delta}(\mathbf{x}_0,t\text{-}t_0/f\text{-}f_0)$ denotes a unit vector with a 1 at the position corresponding to voxel $\mathbf{x}_0$ and time/frequency t_0/f_0. The zoomed area shows the transition between regular (2-fold) and random undersampling.

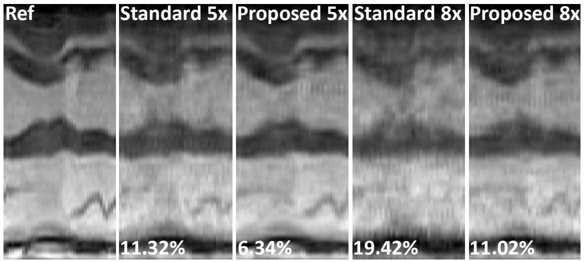

Figure 2 Temporal profiles along the indicated line in Fig. 3. The proposed sampling scheme shows improved unfolding and temporal resolution compared to standard CS sampling. NRMSE of reconstructed profiles are also depicted.

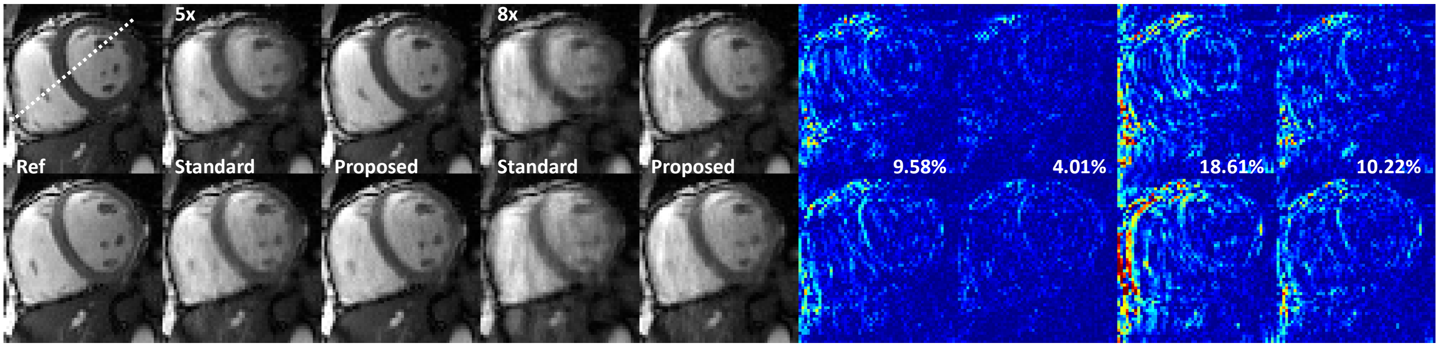

Figure 3 Systolic (top row) and diastolic (bottom row) reference and reconstructed images from 5- and 8-fold undersampled data (Standard CS sampling: left, proposed sampling: right). Magnitude error maps depict error reduction due to combination of coherent and incoherent undersampling. Normalized root mean square errors are quoted.

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