# Randomized Golden Ratio Sampling For Highly Accelerated Dynamic Imaging

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**Introduction:** Transform sparsity is a useful constraint for denoising and reconstructing certain types of undersampled MRI data [1]. Recently, several groups have shown that clinical MR angiography and dynamic contrast enhanced (DCE) imaging can be performed using vastly undersampled 4D k,t-space acquisitions, with reconstruction that utilizes sparsity constraints and parallel imaging [2,3]. Prior work has adopted variable density Poisson-disc (PD) sampling, which is computationally inefficient, and golden ratio (GR) radial-Cartesian sampling [4-8], whose sampling pattern can be adjusted to increase incoherence in the undersampled images. This work explores the potential advantages of randomized golden ratio (RGR) radial-Cartesian sampling, a simple adaptation of GR, for highly accelerated DCE-MRI.

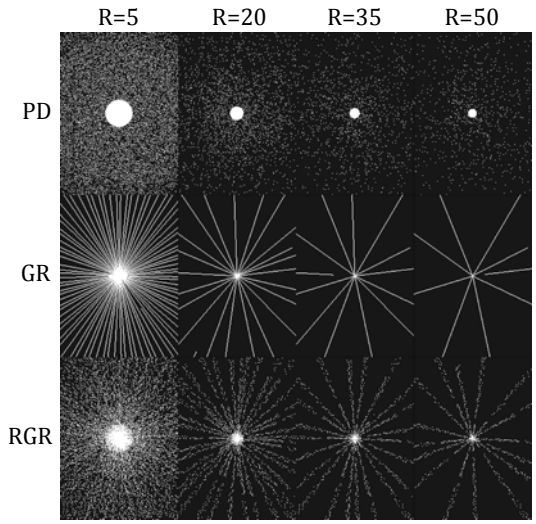


Figure 1 Sampling patterns of Poisson-disc (PD), golden ratio (GR), and randomized GR (RGR) at acceleration rate R = [5, 20, 35, 50].

**Methods:** The sampling pattern in this work is based on GR-CAPR [8]. We introduce (a) a small random perturbation angle ($P_a$) for each phase encode (PE); (b) a probability for sampling each PE ($P_s$), except for those PE's within a defined central region; (c) a temporal window within which PE's are not repeated, defined as a fraction ($W_t$) of the temporal resolution. RGR parameters were chosen empirically: $P_a=\pm 0.01\pi$; $P_s=0.6$, center regions similar to those of PD which has 15% of the PE's in the center; $W_t=0.5$. With these parameters, the RGR sampling table for a 10-min scan can be generated in 100 ms.

DCE data of three brain tumor patients were acquired with 3T GE HDxt scanner using 8-channel head coil and T1w 3D SPGR sequence with 15° flip angle and 5.5 ms TR. Parallel imaging was not used. The fully-sampled data had a 256×186×10 matrix size and 35 time frames; The spatial and temporal resolution were 0.93×1.3×6.0 $mm^3$ and 10 s, respectively.

We reduced data dimensionality by using one central slice from inverse Fourier transform of 4D raw data along $k_z$, and retrospectively sub-sampled in the $k_x$-$k_y$ plane with PD, GR and RGR schemes at four acceleration rates (R) between 5× and 50×. We generated 50 sampling patterns for each method, by altering the pseudorandom generator seed (PD, RGR) and initial angles (GR). Fig. 1 shows the samples for one time frame for each method. Images were reconstructed using alternating direction method of multipliers (ADMM) [9], and analyzed using normalized root-mean-squared-error (nRMSE) compared to the fully-sampled reference. Mean nRMSE and standard deviations were calculated from each subject, and then averaged.

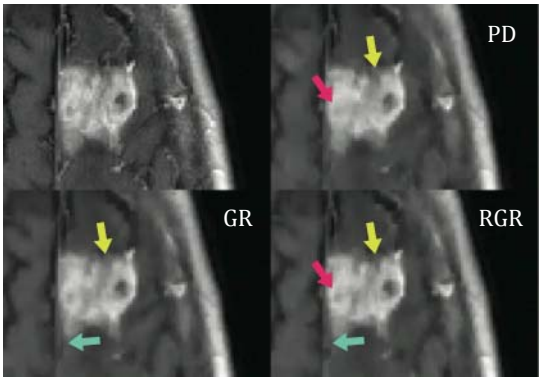


Figure 2 Brain tumor reconstructed (R=50) from PD, GR, RGR and fully sampled data. Arrows are color-coded to show finer details with RGR in different comparisons with PD and GR.

**Results:** Fig. 2 demonstrates representative images, cropped to show detail in the tumor, from fully sampled data, and R=50 undersampled data using PD, GR and RGR. Increased detail is observed with RGR sampling (arrows) relative to the other methods. In all cases, nRMSE increased with acceleration rate but the proposed RGR method reconstructed images with lowest nRMSE at high acceleration rates, Fig. 3. Standard deviations were extremely small compared with mean nRMSE, suggesting that initial conditions (like spoke angle and random number generator seed) are inconsequential.

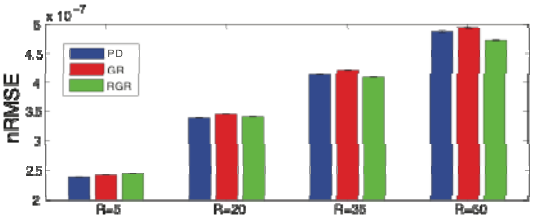


Figure 3 Normalized root-mean-squared-error (nRMSE) and standard deviations derived from PD, GR and RGR results.

**Discussion:** The proposed RGR is novel method to generate randomized sampling scheme combining properties of PD and GR, and provides lower nRMSE in high acceleration cases. RGR can be on-line generated, using parameters such as $P_a$, $P_s$ and $W_t$ that can be tuned to alter different degrees of randomness. Further extensions and benefits of RGR are currently being explored and evaluated.

**Reference**:[1] Lustig, et al., MRM:58:1182-1195, 2007; [2] Lebel, et al., MRM, 2013; [3] Zhang, et al., ISMRM, p2624, 2013; [4] Winkelmann, et al., TMI:26:68-76, 2007; [5] Doneva, et al., ISMRM, p641, 2011; [6] Song, et al., MRM:44:825-832, 2000; [7] Vogt, et al., ISMRM, p92, 2007; [8] Haider, et al., MRM:60:749-760, 2008; [9] Ramani, et al., ISBI, p385-388, 2011.