

**Introduction:** Single-shot echo-planar imaging (EPI) is commonly assumed to be immune to subject motion. However, head movements can reach velocities in the 100mm/s or 100°/s range [1], resulting in translations and rotations of several mm or degrees over the course of a typical readout train (tens of ms). Therefore, a simulation was performed to determine the effect of such fast movements on EPI scan quality. The phase effects due to motion during the readout are expressed in terms of a k-space variable which allows for correction during image reconstruction.

**Theory:** The acquired phase  $\Phi$  of a spin at position  $\mathbf{r}(t)$  due to the influence of a temporally varying magnetic field  $\mathbf{g}(t)$  can be calculated by integrating incremental phase changes:  $\Phi(t, \mathbf{r}) = \gamma \int_0^t \mathbf{g}(t') \mathbf{r}(t) dt'$  (1). For rigid body motion, the motion trajectory of a spin can be written as  $\mathbf{r}(t) = \mathbf{R}(t) \mathbf{r}_0 + \mathbf{d}(t)$  (2), where  $\mathbf{r}_0$  is the initial spin position,  $\mathbf{R}(t)$  is a 3x3 rotation matrix and  $\mathbf{d}(t)$  is the displacement (translation) vector.

Inserting Eq.(1) in Eq.(2) yields:  $\Phi(t, \mathbf{r}) = \gamma \int_0^t \mathbf{g} \cdot (\mathbf{R} \mathbf{r}_0 + \mathbf{d}) dt' = \gamma \int_0^t \mathbf{g} \mathbf{R} \mathbf{r}_0 dt' + \gamma \int_0^t \mathbf{g} \mathbf{d} dt'$  (3).

Eq.(3) shows that the effects of translations and rotations are independent. The phase change due to translations is global (does not depend on  $\mathbf{r}_0$ ), whereas rotation-induced phase effects depend on the spatial position. We split the rotation matrix into a static and a dynamic component:  $\mathbf{R} = \Delta \mathbf{R}(t) \cdot \mathbf{R}_0$ . Without loss of generality we can neglect the static component and set  $\mathbf{R}_0 = 1$ . For small rotation angles, we can approximate  $\Delta \mathbf{R}(t) \approx 1 + \mathbf{N} \omega t + \mathbf{N} \alpha t^2$  (4), with the skew symmetric matrix rotation  $\mathbf{N}$  (which corresponds to the cross product with the stationary rotation axis  $\mathbf{n}$ ) and the angular velocity and acceleration coefficients  $\omega$  and  $\alpha$

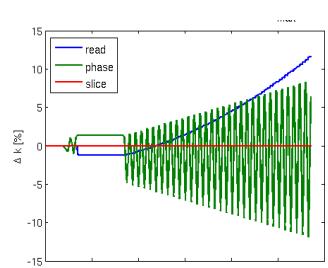


Figure 2: K-space distortion for an angular velocity of 100 deg/s in % of kmax

effective k-space evolution due to rotational effects  $\mathbf{k}_{rot} = \mathbf{R}_0^{-1} \mathbf{k}^{(0)} + \mathbf{N} \omega \mathbf{k}^{(1)} + \mathbf{N} \alpha \mathbf{k}^{(2)}$  (5), where  $\mathbf{k}^{(n)} = \int_0^t t^n \cdot \mathbf{g}(t') dt'$  is the n<sup>th</sup> trajectory moment. Thus, the phase as a function of time and the initial position is  $\Phi_{rot}(t, \mathbf{r}) = \Phi_{rot}(\mathbf{k}, \mathbf{r}) = 2\pi \mathbf{r}_0 \cdot \mathbf{k}_{rot}$ .

**Translation effects:** We obtain a spatially constant phase:  $\Phi_{trans}(t, \mathbf{r}) = \Phi_{trans}(\mathbf{k}) = 2\pi [d_0 \mathbf{k}^{(0)} + \mathbf{v} \mathbf{k}^{(1)} + \mathbf{a} \mathbf{k}^{(2)}]$  (6) with initial position  $d_0$ , constant velocity component  $\mathbf{v}$  and acceleration  $\mathbf{a}$ .

**Generalized signal equation:** The MRI signal equation can be generalized to include dynamic rotations and translations  $s(\mathbf{k}) = \int_V \rho(\mathbf{r}) e^{i\Phi_{rot}(\mathbf{k}, \mathbf{r})} d\mathbf{r} \cdot e^{i\Phi_{trans}(\mathbf{k})}$  (7). This makes it possible to simulate the effect of motion during the readout train and provides a way to account for these effects during the image reconstruction step.

**Simulation results:** Simulations of fast in-plane rotations ( $\omega = 100^\circ/s$ , rotation axis in the slice direction) were performed using the EPI trajectory from Fig.1 with FOV=192x192 mm<sup>2</sup> and matrix= 64x64. The acquired signal in the presence of a fast head rotation during the readout was calculated using Eq.(7) with  $\mathbf{k}_{rot}$  given by Eq.(5). The effect on the final image was simulated by performing a reconstruction with the nominal (=undistorted) trajectory. Fig. 2 displays the fractional change in k-space position for a constant velocity rotation. The deviation in read direction follows the stepped phase encoding gradient (of the first k-space moment) and the deviation in phase direction resembles that of the oscillating read gradient. The net effect of constant angular velocity and acceleration is a spatially dependent rotation of the k-space trajectory (Fig. 3). Ultimately, these effects result in a rotated image (Fig. 4) that shows some additional, but relatively minor distortion relative to the image without motion.

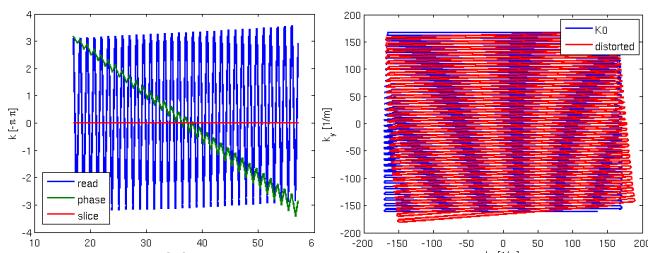


Figure 3: Effective k-space (nominal + distorted) due to an angular velocity component during the readout.

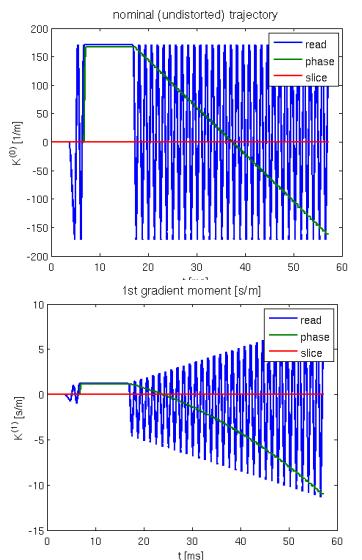


Figure 1: 0<sup>th</sup> (top) and 1<sup>st</sup> – trajectory moment for a 2D-EPI sequence

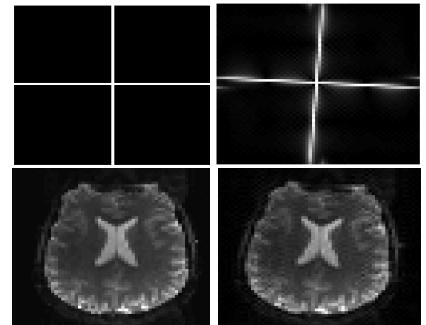


Figure 4: Effect of constant angular velocity (100°/s) over 60ms is a 5° in-plane rotation. Simulations for a cross-shaped pattern (top) and an EPI image (bottom row).

**Discussion:** We show how effects of object rotations during the readout can be formulated in terms of an effective k-space by utilizing higher order trajectory moments. Velocities of 100mm/s and °/s have a relatively minor impact on the quality of lower-resolution EPI scans. The effective rotation angle depends on the accumulated first and 2<sup>nd</sup> order trajectory moment; therefore, the effects will be more pronounced for faster movements, as well as for longer acquisitions (i.e. higher resolutions). Effects are subtle even at these high velocities, but could be important for fMRI/DTI.

#### References

[1] Andrews-Shigaki B, Zaitsev M, Chang L, Ernst T. ISMRM 2009. [2] Nishimura D et al., *Magn Reson Med*, 33, 1995.