MPgCG: An Iterative RF Pulse Design Method for Excitation using Nonlinear Gradient Fields

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Audience: Radiofrequency (RF) pulse designer engineers.

Purpose: Linear gradient fields (LGFs) generate spatial encoding functions (SEFs) that resemble plane waves, and yield an orthogonal set of encoding functions. When nonlinear gradient fields (NLGFs) are used, the SEFs have curved spatial patterns. Inside a practical field-of-view (FOV) such as a rectangular FOV in a Cartesian coordinate system, such SEFs are non-orthogonal to each other, and this may lead to flip-angle variations when used for excitation. Previously, coordinate transformations[1] and iterative methods[2] were used to correct such variations. However, the former method is susceptible to loss-of detail in the target profile (Fig. 1), and the latter requires a predefined k-space trajectory, which may be sub-optimal.

In this study, we propose an alternative pulse design method that uses the Matching-Pursuit[3] (MP) and Conjugate-Gradient[4-6] (CG) algorithms successively, and compare the proposed iterative method to MP and CG as well as non-iterative pulse design methods applied in the Cartesian and nonlinear coordinate systems. In this abstract, the method is demonstrated for a checkerboard target profile (Fig. 2a) and $C2: x^2 - y^2$ and $S2: 2xy$ fields, commonly referred to as PatLoc fields[2]. The simulations are performed at the small-tip-angle regime[7] on the xy-plane, although these are not inherent limitations of the method.

Methods: Simulations are performed using Matlab (Mathworks, Natick, MA) on 40x40 spatial and k-space grids (number of SEFs: 1600). FOV is 20 cm along both x and y, and NLGF amplitudes are adjusted to yield a k-space sampling distance of 1/(20cm)[2]. When LGFs are used, a common approach is to find the excitation k-space using Eq. [1], and discard some of the samples to obtain a practically short RF pulse[7]. However, when the same approach is used with NLGFs, the non-orthogonality of the SEFs in Cartesian coordinates causes flip-angle variations (obtained profile simulated using all SEFs in Eq. [2]), as shown in Fig. 2b. Although such variations can be reduced using the Jacobian, this correction is not perfect since the boundaries of the FOV do not conform to the fields (Fig. 2c). Fig. 2d shows that, although the pulse can be designed using a coordinate transformation, this increases the effective-FOV, and causes loss-of-detail (Fig. 1, Fig. 2d).

The proposed method uses the MP algorithm to select the SEFs iteratively. At each iteration, the SEF that yields the magnitude-wise highest RF value is selected, and its contribution is subtracted from the target profile. However, the non-orthogonality of the SEFs slightly reduces the effectiveness of the MP-algorithm in designing the RF pulse. Therefore, after a predefined number of SEFs are selected, the CG algorithm is utilized to optimize the RF pulse further; hence, the method is referred to as MP-guided-CG (MPgCG).

In addition to the non-iterative methods based on Eq. [1] and coordinate transformations, the method is also compared to CG and MP as well. The root-mean-squared-error[1] (RMSE) is plotted in Fig. 3 with respect to the number of selected SEFs. *Non-iterative methods*: SEFs that yield the magnitude-wise highest RF values are chosen. *CGgCG (CG-guided-CG):* SEFs that yield the highest RF values are chosen after CG is performed using all SEFs. Then, using the selected SEFs, the RF pulse is optimized by running CG again. *MP*: SEF selection and RF pulse design are performed using MP. All profiles are simulated using Eq. [2] using the selected SEFs and the designed RF pulses. Number of CG iterations is 25000 for all CG-based methods.

Results: Fig. 3 shows that, without the Jacobian, the RMSE for Eq. [1] increases as more SEFs are used, due to SEF non-orthogonality. However, the Jacobian reduces the RMSE significantly. Although the coordinate transformation method should theoretically reduce error further, the loss-of-detail (Fig. 1) inhibits its performance. While up-sampling the profile onto a 160x160 grid before transformation reduces the error, it is still higher than the iterative methods, since the method tries to suppress excitation in a larger region (Fig. 1). Although CG and MP yield similar results when LGFs are used (not shown), CG outperforms MP when NLGFs are used and the SEFs are known *a priori*: when all SEFs are used, the RMSE is 30.5% for CG and 38.6% for MP. However, because the SEFs are non-orthogonal in Cartesian coordinates, RMSE increases steeply for CGgCG when some SEFs are discarded. When SEFs are selected using MP, the excitation fidelity is improved significantly. For most cases in Fig. 3, MPgCG yields significantly lower RMSE values compared to the other methods. Although the CGgCG approach yields lower RMSE values than MPgCG for some cases, the RMSE difference is smaller than 0.4% for those and on average, the proposed method yields 20% lower RMSE than CGgCG. Fig. 4 shows the profile obtained using only 400 SEFs with the proposed method.

Discussion: Since the contribution of the selected SEFs is taken into account with each iteration of the proposed method, the obtained profile rapidly converges to the target profile with lower RMSE than the other methods (Fig. 3). The proposed method selects SEFs iteratively, hence, a predefined k-space trajectory is not required. Furthermore, since pulse design is performed in Cartesian coordinates, there is no loss of resolution and the effective-FOV is same as the actual-FOV. It should be noted that the blurriness at the center (Fig. 4) is due to the slow variation of the NLGFs[2], and not an underachievement of the proposed method. Because MP is a greedy algorithm, the selected SEFs may not be the optimal solutions. However, it is shown that the proposed method yields significantly lower RMSE values than the other methods with lower number of SEFs.

After the SEFs are selected, the design of the actual k-space trajectory can be solved using so-called Travelling-Salesman algorithms. Furthermore, in order to limit the length of the trajectory, MP can be constrained to select SEFs that are in a neighborhood of the previously selected ones.

Conclusion: In this study, an iterative RF pulse design approach that uses the MP and CG algorithms is proposed. The method is specifically useful when the spatial encoding functions are not-orthogonal in the computation domain, and may be used for parallel excitation purposes as well.

Acknowledgments: NIH BRP R01 EB012289-01. **References:** **(1)** Kopanoglu, et al., Magn Res Med, 70:537–546 (2013). **(2)** Haas, et al., Magn Res Med, doi: 10.1002/mrm.24559 (2012). **(3)** Mallat and Zhang, IEEE Trans. Signal Process, 41:3397-3415 (1993). **(4)** Hestenes, et al., J Res Nat Bur Stand, 49:409-436 (1952). **(5)** Yip C-Y, et al., Magn Reson Med 2005;54(4):908-917. **(6)** Grissom W, et al., Magn Reson Med 2006;56(3):620-629. **(7)** Pauly et al., J Magn Reson, 81:43-56 (1989).

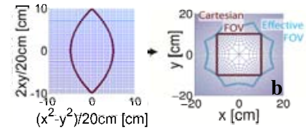


Figure 1: **(a)** A square FOV of 20cmx20cm on the xy-plane corresponds to a sub-region on the C2-S2 plane (red contour). **(b)** Such a coordinate tranformation creates an effective-FOV on the xy-plane that encloses the actual-FOV (Cartesian-FOV). In the effective-FOV, only 33% of the samples represent the actual-FOV, which may lead to a loss of detail in the target profile. Furthermore, a non-iterative pulse design method will try to suppress the excitation inside the whole effective-FOV, which, while reducing the error outside the actual-FOV, may increase the error inside.

$$\boldsymbol{M}_{\kappa} = \sum_{p \in FOV} \boldsymbol{m}_p \cdot \exp\left(i2\pi \boldsymbol{k}_{\kappa} \boldsymbol{f}_p\right) \Delta \boldsymbol{x}_p \quad [1]$$

$$\boldsymbol{m}_p^{obt} = \sum_{\kappa \in W_k} \boldsymbol{M}_{\kappa} \cdot \exp\left(-i2\pi \boldsymbol{k}_{\kappa} \boldsymbol{f}_p\right) \Delta \boldsymbol{k}_{\kappa} \quad [2]$$

$\boldsymbol{m}$: target excitation profile. $\boldsymbol{M}$: k − space representation of $\boldsymbol{m}$.
$\boldsymbol{f}$: spatial variation of NLGFs. $\boldsymbol{p}$: index of spatial coordinates.
$\boldsymbol{\kappa}$: index of k − space. $\boldsymbol{FOV}$: field of view, same along x and y
$\boldsymbol{W_k}$: k − space window width, same along k_{c2} and k_{s2}

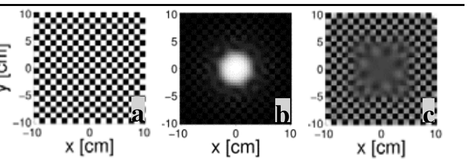
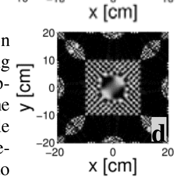


Figure 2: **(a)** Target excitation profile.**(b)** Profile obtained using Eqs. [1] and [2] has significant flip-angle variations. **(c)** Using the Jacobian in Eq. [1] reduces flip-angle variations. **(d)** Coordinate-transformation based method tries to suppress the excitation in a larger region [FOV: (40cm)[2]].



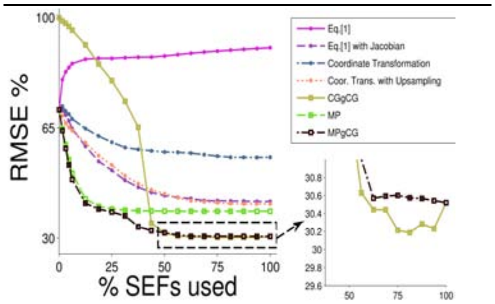


Figure 3: RMS error plotted with respect to the number of selected SEFs (max. number of SEFs: 1600).

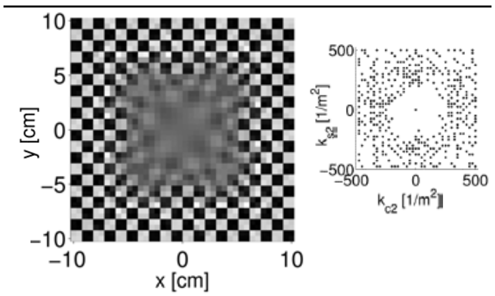


Figure 4: Left: Simulated excitation profile for the RF pulse designed using the proposed method with 400 SEFs. RMSE: 37.5%. **Right:** Locations of selected SEFs in k-space.