

Dynamic Noise Reduction in MRI

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Purpose: Reducing the measurement noise but also preserving object boundaries and fine anatomical structures is a central problem in MRI. A simple approach for dynamic data is to consider each voxel in isolation and reduce its noise using standard temporal smoothing techniques. But applying the same noise reduction scheme to all voxels removes interesting physiological features such as small blood vessels in the denoised images, which defies the whole purpose of denoising. We could do better by considering that voxels belonging to physiologically distinct regions have distinct static and dynamic intensities. Topographical averaging utilizes this idea to reduce noise by averaging signal intensities among neighboring voxels. But such low-pass filters blur tissue boundaries as in Fig1 since in reality many of the neighboring voxels do not belong to the same tissue type. In addition, the computational cost of these approaches are very high for MR data with dimensions of $\sim 10^5$ for static or $\sim 10^7$ for dynamic images. Here, we provide a noise reduction technique that is conceptually simple like topographical averaging methods, computationally efficient, and automatically sensitive to regional boundaries. We further apply our method to DSC MRI of a brain.

Target Audience: Image analysts and image processing scientists.

Methods: Let Y be the $N \times T$ dimensional data matrix where N is the number of voxels and T is the number of temporal samples. Its denoised version \hat{X} estimates the true noiseless X . Topographical smoothing of dynamic data involves,

$$\hat{X} = \arg \min_X \|Y - X\|_2^2 + \lambda_s \|D_s X\|_2^2 + \lambda_t \|D_t X\|_2^2 \quad (1)$$

The first term ensures that the estimate is similar to the observation. The second and third terms respectively penalize too much of spatial and temporal difference between neighboring voxel and neighboring time points, and basically favor spatiotemporal smoothness. Technically speaking, matrices D_s and D_t are gradients in the spatial and temporal domains. The regularization or tuning parameters λ_s and λ_t balance the smoothness terms versus fitness with respect to the data. If they are zero, the estimate is the same as the observation. As λ_s increases, our estimate becomes smoother and boundaries are blurred as more neighboring voxels are averaged. This method assumes that all neighboring voxels have similar dynamic responses grow smoother as in Fig2. This method assumes that all neighboring voxels have similar dynamic responses across tissue boundaries.

Our method, however, averages neighboring voxels according to their physiological regions. For example, it will not average two voxels at tissue boundaries where one voxel pertains to a blood vessel and its neighbor to grey matter. Our method involves the following optimization,

$$\hat{X} = \arg \min_X \|Y - X\|_2^2 + \|\Lambda_s D_s X\|_2^2 + \lambda_t \|D_t X\|_2^2 \quad (2)$$

where Λ_s is a diagonal matrix with zeros or λ_s on the diagonals. It controls whether a voxel should be averaged with its neighbor. To construct Λ_s , we produce the spatial gradient map of the image. It acts as an edge detector and is constructed by subtracting each voxel intensity from its neighbor. If the difference of two voxels is significantly larger than the typical value of noise, then they belong to different physiological regions and should not be averaged. But if the difference is comparable to noise, the voxels belong to the same group and spatial averaging reduces their noise. From here we can see that the zeros on the diagonal of Λ_s correspond to voxels with neighbors of a different tissue type, whereas the nonzero diagonals coincide with voxels with similar neighbors. This idea is inspired by Donoho's soft thresholding approach to denoising¹. Another advantage of our method is that regularization parameters λ_s , λ_t are estimated from the data itself. We marginalize the likelihood over X to let the data tell us about the tuning parameters. This method is known as the empirical Bayes or marginal likelihood. Taking advantage of the fast matrix inversion techniques for sparse and banded matrices significantly reduces the computational cost of these optimizations.

MRI: GE EPIs of Gd-DTPA administered at a dose of 0.1mmol/kg and rate of 5mL/s were acquired at 1s intervals for first 60s, and at 5s intervals for next 300s totaling 120 samples. Imaging was performed on a 3T Siemens whole body scanner with an 8ch phased array head coil. Parameters: TR=1000ms, TE=32ms, 10 contiguous 3mm thick axial slices, matrix 128×128, FOV=220×220mm², FA=30°, BW=1396Hz/voxel, in-plane resolution 1.7×1.7mm².

Results: The topographical smoothing of Eq (1) reduces noise but over-smoothes the image. Relevant structures and tissue boundaries are lost as seen in Fig1. The optimal tuning parameters are chosen from the data using the empirical Bayes method, and according in Fig2, a larger one culminates in overly smooth dynamics. According to Fig3, our method preserves the boundaries while reducing noise. The reduced noise is in the order of the typical value of background noise ~11. Very little structural information is present in the difference image of Fig3 compared to Fig1.

Discussion: This formulation is a batch method, which estimates the dynamics from the whole history of the data and couples temporal changes of voxels so that most voxels will tend to have similar dynamics. Our spatial penalty incorporates the four horizontally and vertically adjacent voxels whereas a more sophisticated formulation can include the other four diagonally adjacent ones.

Conclusions: Spatiotemporal noise reduction is an important tool for data analysis. We presented a noise reduction technique in which our prior belief of spatiotemporal smoothness of the data is expressed in terms of averaging relevant voxels based on which tissue type they belong to, and the weight placed on this smoothness prior is automatically determined from the data itself. Conceptual simplicity and the computational speed of this method are among its other advantages.

Reference: [1] Donoho, D.L. "De-noising by soft-thresholding." *Information Theory, IEEE Transactions on* 41.3 (1995): 613-627.

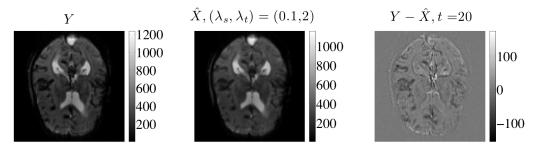


Fig1: The actual image Y , its noise reduced version using simple topographical averaging and the difference between the two is shown for the 20th acquisition. Some boundaries are not preserved.

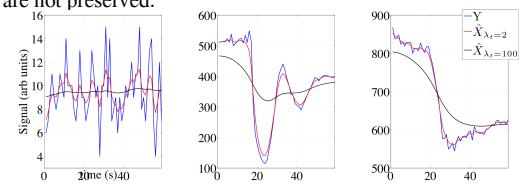


Fig2: Noise is reduced with different temporal tuning parameters.

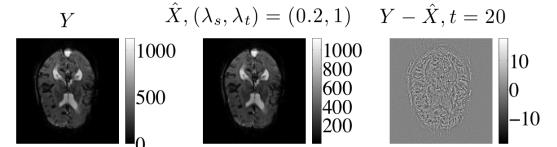


Fig3: The same slice of Fig1 is shown after the application of our algorithm.