Estimating T1 from Multichannel Variable Flip Angle SPGR Sequences with Graph Cuts

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Target Audience: MR reconstruction specialists

Purpose: Quantitative measurements of T_1 values have been used to monitor pathology[1] and have yielded more sensitive detection of pathological states (e.g. myocardial edema[2]). Commonly used T_1 estimations techniques do not account for noise and produce error prone T_1 estimates with limited accuracy and repeatability. Recently a maximum likelihood estimator[3] (MLE) was created to obtain the best estimate of T_1 values in the presence of noise, with no *a priori* assumption about image structure, for a variable flip angle spoiled gradient-recalled echo sequence (VFA-SPGR). This was recently extended for the multichannel case.[4] While this technique showed improvements of T_1 estimation over methods which do not explicitly account for noise, individual voxel estimates nonetheless exhibit high variance. Since the brain is comprised of homogenous regions of tissue, it is believed that each region should have similar T_1 values. It is hypothesized that adding a spatial prior penalizing adjacent voxels with differing T_1 values will further improve the quality of estimation. In this study, a spatial prior that promotes piecewise smoothness was added to the previously developed cost function. Since standard continuous optimization techniques (e.g., Gauss-Newton) are inefficient and unstable for this class of generalized nonlinear least squares problems, an iterative graph cut strategy was developed to provide a robust mechanism for performing regularized T_1-estimation

Methods: Given an image of $x \in N_x$ voxels, the signal from a VFA-SPGR sequence with N_i flip angles and N_c receiver channels for a given repetition time (TR) can be represented as: $G = F(T_1)M + Z$, where $[F_i(T_1)]_{x,x} = \sin(\theta(x,i))\,(1-\exp(-TR\,/\,(T_1\,(x))))/(1-\cos(\theta(x,i))\exp(-TR/T_1(x)))$, where $F(T_1)$ is a TN_c by N_x matrix, M is a N_x by N_c matrix of complex variables proportional to the voxel's spin density and sensitivity function for the receiver coils, $\theta(i)$ is the effective i^{th} flip angle, and $Z \sim \mathbb{C}\mathcal{N}(0,\sigma^2 I)$, is complex Gaussian noise. B_1 field inhomogeneity was not considered at this time. A cost function was constructed from the likelihood function for G, namely $J(T_1,M) = |F(T_1)M - G|_F^2$, where $|\cdot|_F^2$ is the Frobenius norm. In the MLE approach, the unknown parameters M and T_1 are obtained by minimizing this cost function. Since M is technically unknown but not of interest, it has been demonstrated[3,4] that M can be marginalized out using variable projection[5] (VARPRO) resulting in a function of only T_1, i.e., $J(T_1) = \|[F(T_1)F(T_1)^{\dagger} - I]G\|_F^2$, where † is the pseudo inverse and I is the identity matrix. In this work, a spatial prior, $P = \sum_{x=1}^{N_c}\sum_{n=1}^{N} |T_1(n) - R_1(N(x))|$, where R is a neighborhood about voxel x, was incorporated into the cost function such that $J(T_1) = \lambda P(T_1) + \|[F(T_1)F(T_1)^{\dagger} - I]G\|_F^2$. Neighborhoods were defined as an eight nearest neighbor structure with edges normalized for distance. The scaling parameter λ was manually selected to optimize performance. As previously shown,[6] the minimization of this regularized nonlinear least squares cost function can be carried out by iteratively minimizing a series of binary sub-problems using an iterative graph cut technique. Each sub-problem consisted of allowing each voxel the option of keeping its current T_1 value or changing its value by a prescribed amount (ΔT_1) in order to minimize the cost. At each iteration a new graph was formed. The terminal-weights were assigned using the cost function without the spatial prior where weights to the sink were calculated with the current T_1 value and weights to the source used the T_1 value adjusted by ΔT_1. Edge-weights were assigned using Boykov's formulation[7] in conjunction with the spatial prior. A graph cut was performed on the graph to obtain an optimal decision surface to reassign the T_1 values to minimize the cost given the two choices. T_1 values were updated and this process was repeated until there was no preference to change T_1 values. One healthy volunteer was imaged under an IRB-approved protocol on a 1.5-Tesla system (GE Signa v.14.0 Waukesha, WI) using a standard three-dimensional SPGR sequence (TR/echo time = 15/5 msec, field-of-view = 24 cm, 256 x 256 matrix, BW = ±31.25 kHz, NEX = 1, 28 1 mm axial slices) with an 8 channel head coil and five flip angles (5°, 10°, 15°, 20°, 25°). Reconstruction of a single medial slice was implemented using C++ with OpenMP and min-cut/max-flow operations were performed using Boykov's search tree algorithm.[8,9] All processing was performed on a dual 3.0 GHz Intel Quad-Core Xenon processor computing server (24 MB L2 cache and 32 GB 800 MHz DDR2 memory). In this study, ΔT_1 was alternated between $\pm\Delta T_1$ until there was no change in T_1 values, then ΔT_1 was reduced and the process was repeated. The ΔT_1 decimation schedule was set as [500, 300, 200, 150, 100, 75, 50, 35, 25, 15, 10, 7, 5, 3, 1]ms.

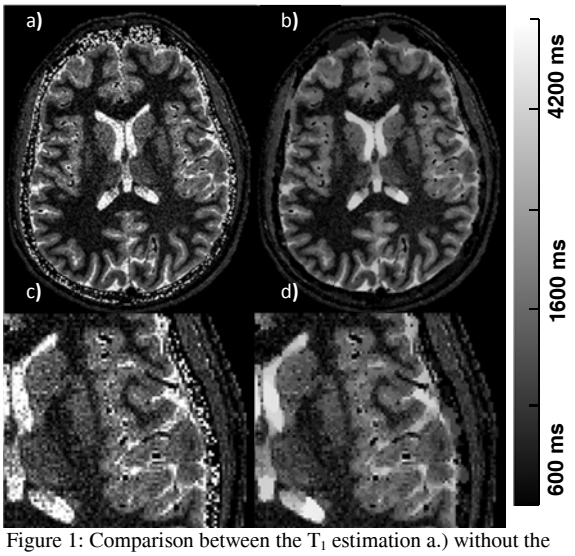

Figure 1: Comparison between the T_1 estimation a.) without the spatial prior b.) proposed method. c-d.) Expansions showing the ventricle, sulci and putamen are illustrated for each technique.

Results: Figure 1 shows the reconstructed T_1 maps of a single slice with and without the spatial prior (λ=0). Expansions are shown to illustrate the reduction of variance in the lateral ventricles, sulci and putamen. Regions of interest were selected to compute mean and standard deviation of white matter (WM), gray matter (GM), lateral ventricle, sulci, and putamen. In the unconstrained reconstruction, low signal – primarily in areas with CSF – resulted in regions with a high T_1 variability. In the proposed method, the mean T_1 of the lateral ventricles was reduced from 29.1e3 ± 81.0e3 ms to 3610 ± 461 ms, and the sulci from 9970 ± 44.0e3 to 2855 ± 593 ms. Minimal changes in mean T_1 values and over a 17% reduction in variance was found in the major tissue classes: GM (1535 ± 325 ms to 1497 ± 219 ms), the putamen (1307 ± 253 to 1271 ± 165), and WM (819 ± 143 ms to 813 ± 117 ms). The total processing time for each image was under 120 sec.

Discussion: The proposed method was able to substantially reduce the variance within the ventricle and the sulci as well as reducing the mean T_1 value closer to expected values for CSF. In regions of high signal, the technique preserved the mean T_1 value while providing a moderate reduction in variance. This is expected as the regularized cost function attempts to estimate most the likely T_1 value for the given signal while simultaneously promoting piecewise smoothness. Despite these advantages, there are still several open problems concerning the proposed approach. At present, the regularization parameter, λ, must be manually assigned, and strongly determines reconstruction performance. Future effort will focus on how to automatically determine the correct value. Computation time also remains a practical barrier with this approach. The primary computational bottleneck of this method lies in execution of the min-cut/max-flow operations on the constructed graphs. Incorporation of recent parallelization concepts for this algorithmic component should substantially reduce this expense.

Conclusion: The iterative graph cut method for minimizing the regularized nonlinear least squared cost function was able to reduce the variance of T_1 values in homogenous regions of tissue while preserving distinct boundaries between differing tissue types. Substantial reductions of variance were shown in the ventricles and sulci, while moderate reductions of variance were found in WM, GM and the putamen with minimal change to the mean T_1 values.

References: [1] Barbosa et al., *Magn Reson Imag* 1994;12:33-42.; [2] Ferreira et al., *J Cardio Magn Reson* 2012; 14:42; [3] Haldar et al., *Proc.. ISMRM* 2007, 41; [4] Trzasko et al., *Magn Reson Med* 2013;69:1787–94; [5] Golub et al., *SIAM J Num Anal* 1973;10:413-32; [6] Hernando et al., *Magn Reson Med* 2010;63:79–90; [7] Kolmogorov et al., *IEEE PAMI,* 2004; 2:147-159; [8] Boykov et al., *IEEE PAMI*, 2004; 9:1124-1137; [9] Boykov et al., (2010). maxflow-v3.01, http://vision.csd.uwo.ca/code/