A Graph Cut Approach to Regularized Harmonic Estimation for Steady-State MR Elastography

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Target Audience: Image reconstruction scientists interested in magnetic resonance elastography.

Purpose: In magnetic resonance elastography (MRE), mechanically induced motion in tissue is estimated from a time-encoded series of phase contrast images [1]. For steady-state MRE, relevant motion information is encoded within the first temporal harmonic (i.e., Fourier coefficient) of this image series. Given an estimate of this quantity (and knowledge of certain scan parameters), quantitative spatial maps of tissue stiffness can be constructed [2]. The accuracy of a tissue stiffness map is intrinsically dependent on the estimated temporal harmonic quantity from which it is derived. Recently, Trzasko and Manduca [3] proposed a robust statistical framework for harmonic estimation that – unlike previous approaches – prospectively accounts for noise in the raw MRI data during the estimation process. Despite promising initial results, the gradient-based numerical optimization strategy employed in [3] for performing harmonic estimation was asserted as viable only for data containing no phase wrap. That restriction reduces the practicality of this approach. In this work, we describe an alternative optimization strategy based on graph cuts [4] that can effectively perform robust harmonic estimation in the presence of wrapping. Unlike prior applications of graph cuts for quantitative MRI (e.g., field map estimation in fat+water imaging [5]), the target estimation quantity here is complex-valued. This raises several unique computational challenges (and solutions), which are discussed below.

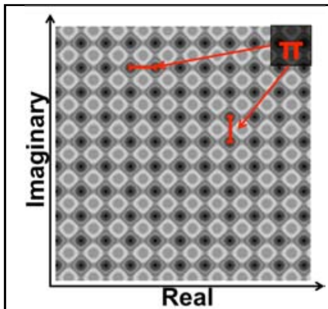

Fig. 1) Complex solution space (J) of a single voxel (*T*=4). Dark values represent local minima.

Methods: In a standard steady-state MRE acquisition, the signal observed by receiver channel $c \in [0, C-1]$ during phase offset $t \in [0, T-1]$ at spatial position x (out of N voxels) for a single motion-encoded direction can be modeled as $\boldsymbol{G}_{\pm}[x,t,c] = \boldsymbol{M}[x,t,c]\exp(\pm \boldsymbol{j}\mathrm{Re}\{\boldsymbol{\eta}[x]\boldsymbol{\zeta}[t]\}) + \boldsymbol{Z}_{\pm}[x,t,c]$**,** where $\pm$ denotes the polarity of motion encoding, $\boldsymbol{M}$ is a potentially time-varying background signal (e.g., due to intra-voxel phase dispersion (IVPD) [6]), $\boldsymbol{\eta}$ is the target harmonic quantity, $\boldsymbol{\zeta}[t] = \exp(j2\pi t/T)$, and $\boldsymbol{Z}_{\pm}[:,:,\cdot] \sim CN(\boldsymbol{0}, \boldsymbol{\Psi})$ is complex Gaussian noise such that $\boldsymbol{\Lambda} = \mathrm{E}[\boldsymbol{Z}^{*}\boldsymbol{Z}] = 2NT\boldsymbol{\Psi}^{\boldsymbol{T}}$. This signal can be expressed as an $NT \times C$ matrix, $\boldsymbol{G}_{\pm} = \boldsymbol{H}_{\pm}(\boldsymbol{\eta})\boldsymbol{M} + \boldsymbol{Z}_{\pm}$, where $[\boldsymbol{H}_{\pm}(\boldsymbol{\eta})]_{(x,t),(x,t)} = \exp(\pm \boldsymbol{j}\mathrm{Re}\{\boldsymbol{\eta}[x]\boldsymbol{\zeta}[t]\})$ is diagonal, and the set of all measurements as a $2NT \times C$ matrix $\boldsymbol{G} = [\boldsymbol{G}_{+}^{T} \quad \boldsymbol{G}_{-}^{T}]^{\boldsymbol{T}} = \boldsymbol{H}(\boldsymbol{\eta})\boldsymbol{M} + \boldsymbol{Z}$. Robust harmonic estimation comprises identifying $\boldsymbol{\eta}$ which minimizes a cost functional constructed of a penalty and the marginalized negative log-likelihood of $\boldsymbol{G}$, i.e., $\boldsymbol{J}(\boldsymbol{\eta}) = \boldsymbol{\lambda}\boldsymbol{P}(\boldsymbol{\eta}) - \sum_{x,t}|\boldsymbol{K}[x,t]|\cos(2\mathrm{Re}\{\boldsymbol{\eta}[x]\boldsymbol{\zeta}[t]\} - \measuredangle\{\boldsymbol{K}[x,t]\})$, where $\boldsymbol{K}[x,t] = \sum_{c,d}\boldsymbol{\Lambda}^{-1}[c,d]\,\boldsymbol{G}_{+}[x,t,c]\overline{\boldsymbol{G}_{-}[x,t,c]}$ and $\boldsymbol{\lambda} \geq 0$ is a user-selected regularization parameter. As in [3], we adopt a quadratic smoothness penalty, $\boldsymbol{P}(\boldsymbol{\eta}) = \sum_{x,b\in\Theta(x)}|\boldsymbol{\eta}[x] - \boldsymbol{\eta}[b]|^{2}$, where $\Theta(x)$ is some spatial neighborhood of x. The computational challenge of this estimation problem is due to the presence of the cosine term in $\boldsymbol{J}(\boldsymbol{\eta})$, which is 2π-periodic (and thus non-convex). It follows that $\boldsymbol{J}(\boldsymbol{\eta})$ is invariant to additive perturbations of the harmonic by any $\boldsymbol{\varepsilon}$ satisfying, $\forall x, t$, $\mathrm{mod}(\mathrm{Re}\{(\boldsymbol{\varepsilon}[x])\boldsymbol{\zeta}[t]\}, \pi) = 0$. This is simply a higher-dimensional manifestation of the wrapping phenomenon that occurs in standard phase estimation problems. For example, if $T = 4$, viable perturbations are $\boldsymbol{\varepsilon}[x] = p\pi + jq\pi$, where both p, q are any integers. A plot of $\boldsymbol{J}(\boldsymbol{\eta})$ for $T = 4$ and $\boldsymbol{\lambda} = 0$ over the complex domain of a single voxel is shown in Fig. 1, and highlights the insufficiency of descent-based optimization methods for this problem class. Mirroring [5], the proposed (greedy) optimization strategy operates by executing a series of binary decisions. At iteration i, an update to the current harmonic, $\boldsymbol{\Delta}_{i}$, is proposed and the voxel-wise decision $\boldsymbol{\eta}_{i+1}[x] \in \{\boldsymbol{\eta}_{i}[x], \boldsymbol{\eta}_{i}[x] + \boldsymbol{\Delta}_{i}[x]\}$ is made by forming a structured graph [4] whose min-cut specifies $\boldsymbol{J}(\boldsymbol{\eta}_{i+1}) \leq \boldsymbol{J}(\boldsymbol{\eta}_{i})$. For the cost illustrated in Fig. 1, observe that different basins (i.e., local minima) of the non-convex solution space can be reached via $\boldsymbol{\Delta}_{i}[x] \in \{\pm\pi, \pm j\pi\}$. For the specified $\boldsymbol{P}(\boldsymbol{\eta})$, it can be shown (proof omitted for brevity) that each binary decision problem is graph representable [4] if all elements of the complex update, $\boldsymbol{\Delta}_{i}$, have the same phase. This is akin to the requirement in [5] that all elements of a real-valued field map update be of the same sign. The proposed optimization strategy was implemented in C++/OpenMP using the graph construction paradigm in [4], and utilized Boykov et al.'s maxflow-v3.01 software package [8,9] for all graph cut operations. To test the efficacy of the proposed strategy, robust harmonic estimation was performed on an MRE data set of the anal sphincter [7] (2D GRE, 1.5 T, single ME direction, endorectal coil, $T = 4$, $C = 1$) known to contain wrap artifact within individual phase contrast images. A harmonic estimate [2] derived from the temporal Fourier transform of the phase contrast sequence was used as an initializer. For this experiment, $\boldsymbol{J}(\boldsymbol{\eta})$ was defined with $\boldsymbol{\lambda} = 2500$ and, $\forall x$, with $\Theta(x)$ comprising the 4 cardinal neighbors of x. The following schedule was used for the magnitude of the (complex) harmonic updates: {π, π/2, π/4, π, π/8, π/16, π/32, π, π/64, π/128, π, π /128}. For $|\boldsymbol{\Delta}_{i}| = \pi$ updates, $\measuredangle\{\boldsymbol{\Delta}_{i}\}$ was chosen sequentially from $\{0, j\pi/2, j\pi, j3\pi/2\}$ until convergence (defined below); otherwise, $\measuredangle\{\boldsymbol{\Delta}_{i}\}$ was selected randomly in [0,2π]. At a given update stage (fixed magnitude), convergence was assumed when a cut or its binary inverse was observed for 4 consecutive iterations. Our current code implementation requires ~20 min to perform this estimation on a dual 3.0 GHz quad-core server with 32 GB memory. We anticipate that the current estimation times can be greatly reduced through code optimization and parallelization of the graph cut sub-routines.

Results: Fig. 2 shows one raw phase contrast image from the MRE series and the corresponding wave image synthesized from the robust harmonic estimate generated via the proposed graph cut optimization strategy. Observe that the process of robust harmonic estimation simultaneously eliminates phase wrap (arrow) and removes noise without compromising wave information. Additionally, we highlight the absence of false wave information in low signal-to-noise ratio (SNR) regions in the periphery of the robust harmonic estimate image.

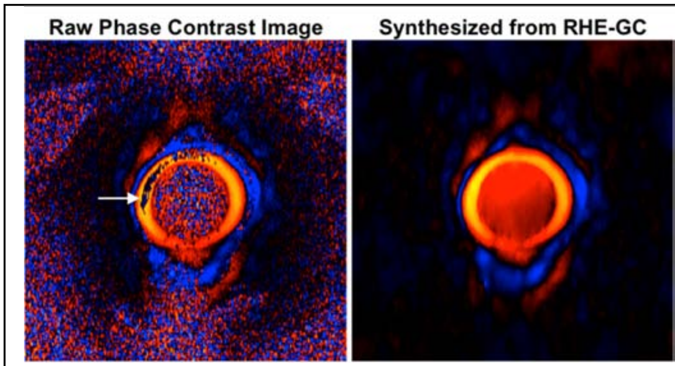

Fig. 2) Single raw image of an MRE (phase contrast) series of the anal sphincter and corresponding wave field synthesized from the graph cut based robust harmonic estimate. Note the absence of phase wrap (arrow) and noise in the graph cut-based estimate.

Discussion: In this work, we have proposed a graph cut based optimization strategy for robust harmonic estimation, which prospectively accounts for noise in raw MRE data during the estimation process. As demonstrated, this approach enables robust harmonic estimation to be performed even in the presence of phase wrap, and thus should both streamline and improve the generation of tissue stiffness maps. Future work will focus on refining the harmonic update schedule to minimize computation time, as well as generalizing this statistical framework to enable direct harmonic estimation from undersampled k-space data (for accelerated applications).

Conclusion: Graph cut based optimization enables robust harmonic estimation for steady-state MRE to be effectively performed even in the presence of phase wrap.

References: [1] Muthupillai et al., Science 269:1854-57, 1995; [2] Manduca et al., Med Imag Anal 5:237-54, 2001; [3] Trzasko and Manduca, ISMRM 2012:3425; [4] Kolmogorov et al., IEEE PAMI 26:147-159, 2004; [5] Hernando et al., MRM 63:79–90, 2010; [6] Glaser et al., MRM 50:1256-65, 2003; [7] Kruse et al., ISMRM 2012: 406; [8] Boykov et al., IEEE PAMI 9:1124-37, 2004; [9] http://vision.csd.uwo.ca/code/