Complementary Poisson-Disc Sampling

Evan Levine[1,2], Manojkumar Saranathan[2], and Brian Hargreaves[2]
[1]*Electrical Engineering, Stanford University, Palo Alto, CA, United States,* [2]*Radiology, Stanford University, Palo Alto, CA, United States*

Purpose: Many 3D dynamic MRI sequences use stochastic k_y-k_z-t sampling trajectories for multiphasic imaging.[1,2,3] Some approaches use incoherent sampling in both *k* and *t* domains and accommodate smooth variable density *k*-space sampling profiles. Poisson-disc *k*-space sampling is commonly used to allow sampling incoherence for compressed sensing (CS) but with uniformity to control g-factor in parallel imaging.[1] In other approaches such as DISCO, dynamic phases use pseudo-random sampling, and view-sharing is used to form composite images that can be reconstructed with non-iterative parallel imaging.[2,3] DISCO achieves a pseudo-random sampling distribution by sorting samples within annular regions in radial distance, undersampling by N, and distributing them across N temporal phases. Both stochastic trajectories allow individual phases to be reconstructed with iterative methods based on parallel imaging and CS and entail similar tradeoffs in spatial and temporal resolution. The pseudorandom distribution used in DISCO is complementary but not ideal for CS, while conventional Poisson-disc sampling is not complementary in time. To unite the advantages of both, we have developed a method for designing k_y-k_z-t trajectories consisting of complementary pseudo-random phases with Poisson-disc sample distributions. This method can allow both CS reconstruction from Poisson-disc sampling and non-iterative parallel imaging reconstruction of view-shared composite images.

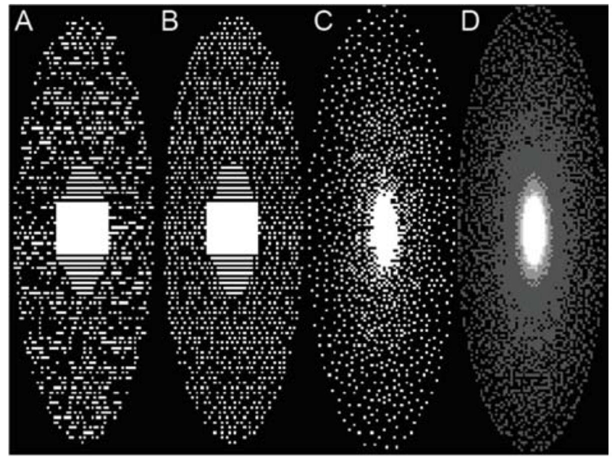

Figure 2: Pseudorandom sampling distributions of representative individual phases of standard DISCO (A), Poisson DISCO (B) and variable density Poisson-disc (C). The sum of variable density phases (D) shows that sampling is complementary.

Methods: The method extends the Poisson-disc sampling procedure to repeatedly draw a sample from a probability density function over a subdomain of k_y-k_z-t space conditioned on a minimum distance required between samples, then periodize the sample in time. Minimum distance criteria include a temporal distance, which is fixed to guarantee complementary sampling, and a *k*-space distance, which is reduced whenever it is too large to permit new samples. The procedure stops when all temporal phases are filled. We employed the technique in two examples, a new trajectory for DISCO and a similar variable density sampling pattern.

Example 1: "Poisson DISCO:" Samples were generated within an annular region and on a rectangular grid with two-fold undersampling in the phase-encoding direction. Three phases filled the annular region completely and without overlapping samples, and the central region was fully-sampled in each phase. Poisson-disc sampling distributions were visually compared with those of standard DISCO. Liver images were retrospectively undersampled using both patterns and reconstructed with ESPIRiT for parallel imaging. [4] Both patterns included twofold regular acceleration in the phase-encoding direction.

Example 2: Variable density sampling: Samples were generated on a Cartesian grid and within an elliptical region using a probability density function of the form $p(k) = Z / \|k\|_2$ to produce a variable density profile. Samples were periodized with period proportional to p(k).

Results and Discussion: Pseudorandom Poisson-disc sampling patterns (Fig. 1B-C) have more even sample distributions than those of standard DISCO (Fig. 1A) but all have dispersed point spread functions (result omitted for clarity). The sum of four variable density phases (Fig. 1D) show that they are complementary in the same sense— the degree of overlap between phases is strictly decreasing with *k*-space radius in proportion to the sampling density of individual phases.

CS reconstruction of individual phases in DISCO is greatly improved with the proposed sampling pattern (Fig. 2). Differences might be expected when parallel imaging is used conservatively for composite images and provides additional benefit for the individual phases.

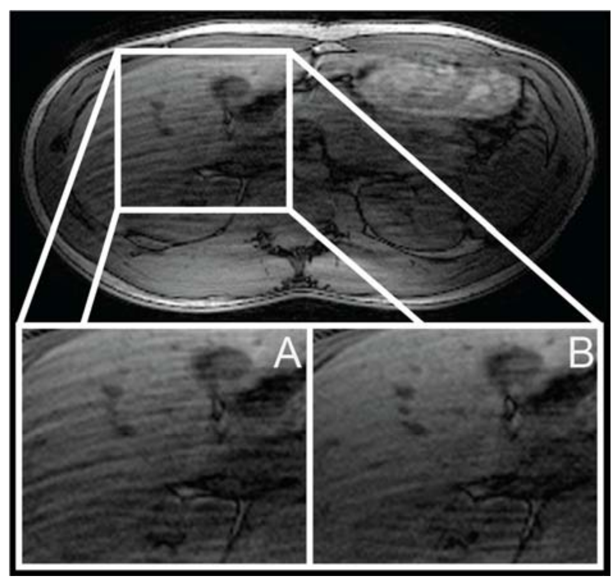

Figure 2: Liver images reconstructed from individual phases of standard DISCO (A) and Poisson DISCO (B) show an improvement in reconstruction quality with Poisson-disc sampling.

Conclusion: Complementary Poisson-disc sampling judiciously unites the advantages of incoherent, uniform, and complementary *k*-t sampling, enabling acceleration from new combinations of CS, parallel imaging, and view-sharing. For DISCO, complementary temporal sampling is maintained while spatial sampling is improved, and for variable density Poisson-disc sampling, spatial sampling is maintained while temporal sampling is improved.

References: [1] Lustig et al. MRM 2010; 64:457-71 [2] Saranathan et al. JMRI 2012; 35:1484-92 [3] Song et al. MRM 2009;61:1242–48 [4] Uecker et al. MRM 2013