Uniformity and symmetry considerations in sampling designs for 3D radial MRI

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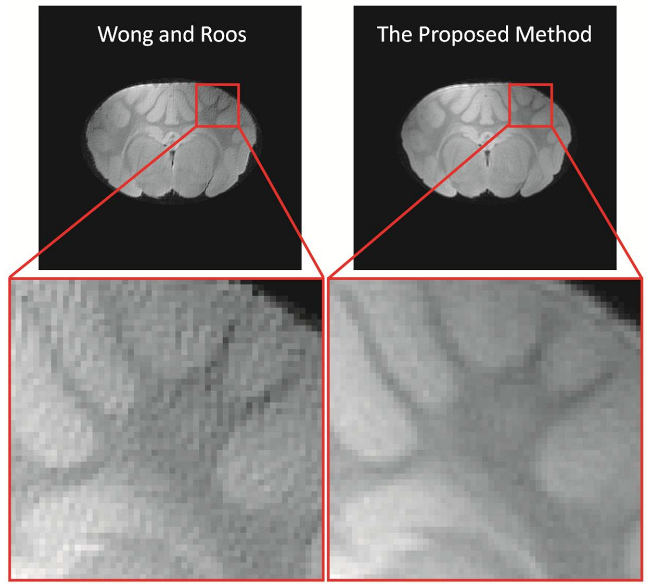
TARGET AUDIENCE: Clinicians and researchers who use 3D radial MRI

PURPOSE: Uniformity in sampling designs for 3D radial MRI is well known since many works, e.g. [1-3], have been built upon either the works of Wong and Roos [4] or Saff and Kuijlaars [5] to achieve nearly uniform sampling on the unit sphere. Here, we argue that further improvement in image quality can be achieved by taking antipodal symmetry (or Hermitian symmetry) into account in the sampling design process. To test the above hypothesis, we developed a completely novel technique capable of generating very large, uniformly distributed points on the unit sphere that are endowed with antipodal symmetry.

METHODS: The approach is based on centroidal Voronoi tessellations [6] on the sphere with a novel pseudometric [7] and a new strategy for accelerating the iterative process of centroidal Voronoi tessellations. Current state-of-the-art technique for tessellating the unit sphere, hereafter denoted as Algo A, is O(n log n) where n is the number of points on the sphere. Conventional centroidal Voronoi tessellations (CCVT) would entail invoking Algo A m number of times, i.e., CCVT is O(m n log n). Our method takes only O(m n) operations, which implies O(n) per iteration on the average.

RESULTS: Figure 1 shows the reconstructed images of a three-dimensional numerical and realistic phantom constructed from a sheep brain sample with two different schemes, the Wong and Roos method and the proposed method, at the same number of radial projections of 8101 on the upper hemisphere with 128 sampling points along each projection. The reconstructed images obtained through the method of Wong and Roos and the proposed method are respectively, Figures A and B. We found that the reconstructed images from the proposed method to have less streaking artifacts, and are more accurate. Specifically, the average of the percent relative error per pixel (with the brain region) of the method of Wong and Roos and of the proposed method with respect to the ground truth are respectively 30.7% and 21.3%. The reduction in percent of relative error is almost 10%. The improvement in image quality between the method of Wong and Roos and the proposed method is clearly visible in the zoomed in region, see Figure 1. Figure 2 establishes the empirical evidence that the proposed method is O(n) per iteration on the average.



(A) (B)

Figure 1. The central slice of the reconstructed volume by the method of Wong and Roos (Left) and the proposed method (Right).

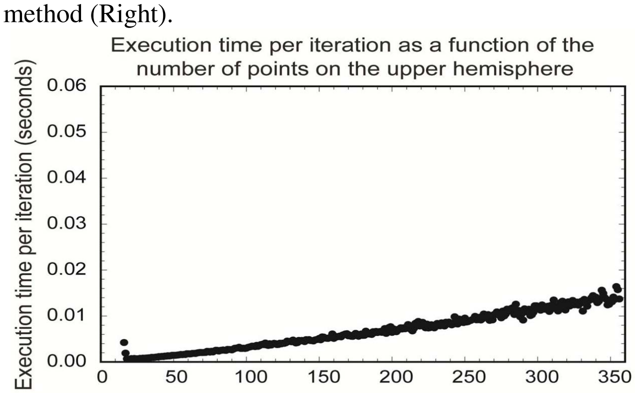


Figure 2. The performance of the proposed method in terms of execution time (in seconds) per iteration of the proposed method as a function of the number of points on the upper hemisphere has a linear trend with respect to n.

CONCLUSIONS: Antipodal or Hermitian symmetry should be taken into account in sampling designs for 3D radial MRI. The proposed method is the first method capable of efficiently and accurately generating very large and very uniform antipodally symmetric points on the sphere.

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