Rangoli Undersampling Library (RUSL) for k-space trajectory design to combine compressed sensing and parallel imaging for accelerated MRI

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Introduction: Compressed Sensing (CS) [1] and Parallel Imaging (PI) [2] are two techniques to speed up the acquisition process in MRI. CS relies on the sparsity in medical images while PI involves combination of the k-space data acquired from multiple channels simultaneously thus reducing the data acquisition time.

Theory: The current work, "Rangoli Undersampling Library (RUSL)" provides a novel framework for quasi-incoherent undersampling patterns based on floral patterns laid using rice-flour called "Rangolis". This is particularly relevant for accelerated acquisitions of 2D applications such as cardiac MR (CMR) or dynamic contrast enhanced MRI (DCE-MRI) with multi-slice imaging or as a single slice acquired over time. Multiple similarities exist between the objectives of the application of magnetic gradients during acquisition, and CS undersampling, and these floral patterns. For instance, some of these patterns are laid out on Cartesian grids but have pseudo-random distribution of points (hybrid space) and are typically connected. Also, the objective of the person drawing these floral patterns is to cover maximum distance along the lines with the amount of flour one has in his/her hand. This is analogous to gradient design where the smooth and large coverage of k-space is desirable. This will provide a library method of choosing an undersampling mask, typically in two (or three) dimensions, to provide increased incoherence while enabling smooth gradient design. The undersampling designs provided by the Cartesian grid based Rangoli designs are ideal for the combination of parallel imaging and compressed sensing based acceleration of MRI as the grid is equi-spaced and allows for a random pattern on this grid.

Methods: Rangoli images were collected from online sites allowing free downloads [3]. Each image was converted to binary image and was resized to 256 X 256. Dots were extracted from rangoli image which acted as k-space trajectory samples and rescaled to axes between -0.5 to 0.5 (kx,ky). A Variable Density Spiral (VDS) was plotted on rangoli dots. Tweaking of the spiral was performed by mapping each VDS point to the nearest rangoli dot by calculating the Euclidian distance between the coordinates without repetition measured by a data consistency error. Consistency error is the difference between dot coordinates and tweaked spiral coordinates and smoothness error is the difference between VDS coordinates and tweaked spiral coordinates. Both the error factors were considered before mapping and were balanced using a regularization factor by varying its value between 0 and 1. If the error obtained was lesser than the previous error, spiral coordinate was mapped to rangoli dot coordinate else the original spiral coordinate was retained. This procedure was followed for all the VDS points to obtain the final trajectory. Point Spread Function (PSF) was created as shown in Figure 1(a) by setting 100 pixels in the center to a value of 1 and its k-space was determined as in Figure 1(b). The PSF was undersampled by collecting samples based on the RUSL trajectory for a given undersampling factor. The required undersampling factor was determined and rangoli image as in Figure 1(c) was thresholded accordingly. Equation $\mathfrak{C} = |\mathbf{R}_c \text{-VDS}_c|_2 + \lambda |\mathbf{TS}_c \text{-VDS}_c|_2$ (1) represents how the error is calculated where $\mathbf{R}_c = \mathbf{R}$ angoli coordinates, $\mathbf{VDS}_c = \mathbf{V}$ ariable Density Spiral coordinates, $\mathbf{TS}_c = \mathbf{T}$ weaked Spiral coordinates and $\lambda = \mathbf{R}$ Regularization factor.

Results and Discussion: A VDS is plotted on a representative rangoli mask as shown in Figure 1(d). Spiral was tweaked following the same procedure as explained above which resulted as in Figure 1(e). It can be observed from the Figure 1(d & e) that by plotting tweaked spiral on rangoli there is more concentration on the rangoli points compared to plotting VDS on rangoli. The resulting undersampled PSF is shown in Figure 1(f) shows incoherent noise and aliasing artifacts which can be removed by using CS and PI reconstruction methods respectively [4]. Figure 2 shows the peak signal to noise ratio (PSNR) values for 6 undersampling factors which show a monotonous decrease as expected. It is also observed from the graph shown in Figure 2 that the PSNR value is not significantly decreasing for increasing acceleration factors. Hence the proposed work can be applied for higher acceleration factors.

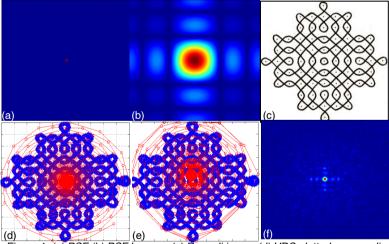


Figure 1: (a) PSF (b) PSF k-space (c) Rangoli image (d) VDS plotted on rangoli (e) Tweaked spiral (f) Undersampled PSF

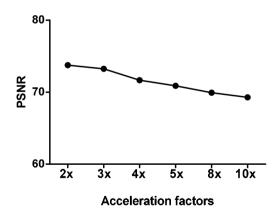


Figure 2: PSNR values for 6 undersampling factors

Conclusion and Future Work: The proposed work comes up with a new scheme for determining k-space trajectory for the combination of CS and PI. Rangoli is ideal for the combination of parallel imaging and compressed sensing based acceleration of MRI as the grid is equi-spaced and allows for a random pattern on this grid. Current and future work involves application of resultant trajectory on MR data obtained from scanner and reconstruction of image using CS and PI.

References[1] Candes EJ et al. IEEE T Inform Theory 2006;52(2):489-509. [2] Pruessmann KP et al. Magnetic Resonance in Medicine 1999;42(5):952-962.[3] www.rangolidesign.net [4] Yoon-Chul Kim et al. IEEE 2009.