Simultaneous Use of Linear and Nonlinear Gradients as Independent K-Space Variables for RF Excitation

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Target audience: Researchers who studies non-linear gradients fields (NGF), SAR reduction and B_1^+ inhomogeneity correction

Purpose: In recent years, non-linear encoding fields are proposed for acquisition[1] and RF excitation[2,3]. Previous studies showed that usage of NGF for RF excitation can reduce SAR[2] and provide $B1^+$ inhomogeneity correction[3]. In this work, we introduce new k-space dimensions for NGF which is totally independent from k-space variables for linear gradients and reestablishes the Fourier Transform relation with increased dimension.This independent dimension provides freedom for trajectory optimization and low energy RF pulse design. On the other hand, to best of our knowledge, no previous study utilized NGF and linear gradients independently for RF excitation. In this work, an example target excitation profile is assumed in order to compensate the effect of example $B1^+$ inhomogeneity. It is shown that using linear and NGF simultaneously for RF excitation can provide more homogenous excitation profiles in comparison with only linear or only NGF.

$$M_{xy}(\mathbf{r}) = i\gamma M_0 \int_{\mathbf{k}} W(\mathbf{k})S(\mathbf{k})e^{j2\pi\mathbf{k}\cdot\mathbf{r}}d\mathbf{k} \quad (1)$$

$$M_{xy}(\mathbf{r}) = i\gamma M_0 \int_{-\infty}^{\infty} L(\mathbf{r},u)\delta(u-u(\mathbf{r}))du \quad (2)$$

$$L(\mathbf{r},u) = \iiiint_{-\infty}^{\infty} W(\mathbf{k},k_u)S(\mathbf{k},k_u)e^{j2\pi(\mathbf{k}\cdot\mathbf{r}+k_u u)}d\mathbf{k}dk_u \quad (3)$$

$$min(\int |f(x) - \Sigma_{n=1}^{N} a_n e^{j2\pi(k_{xn}x+k_{un}u(x))}|^2 dx) \quad (4)$$

Theory: In (1), the relationship between the excitation profile, RF pulse and k-space trajectory is previously derived[4]. Here **r** stands for Cartesian coordinates x, y, z. W is a function of the 3 k-space variables and depends on the ratio of RF pulse and overall speed of the gradient vector. S describes the k-space trajectory in 3D[1]. Starting from the small tip angle approximation and including also a NGF, (2) is obtained where **r** is the Cartesian coordinate variables x, y, z. u(**r**) is the dependence of NGF to the space variables. M_{xy} is the resulting excitation profile. In (3) vector **k** is the k-space variables such as k_x, k_y and k_z. k_u stands for the independent k-space variable for the nonlinear gradient. u is the space variable of the nonexistent 4th dimension. In (3), W is 4D function which is extended version of the variable W in (1) including new dimension k_u. In (3), S is also 4D k-space trajectory which is the extended version of S in (1). We can propose that although L is a 4D function, only its projection onto the 3 regular space variables determines the excitation profile. Although L is an abstract mathematical function of 4 space variables, it would be a valuable design tool. Therefore, we can design function L freely except the points that lie in the projection u= u(**r**). Due to (2), designing L freely allows us to design RF pulse or the k-space trajectory only by taking care of projection of L into 3D.

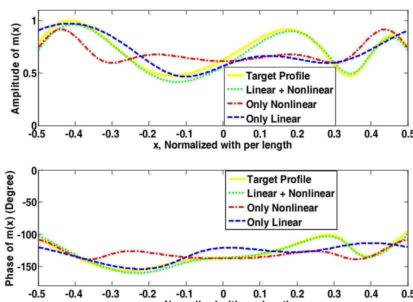

Figure 1: Target excitation profile and simulated excitation profiles are provided for only linear, only nonlinear and combination of linear and nonlinear gradients

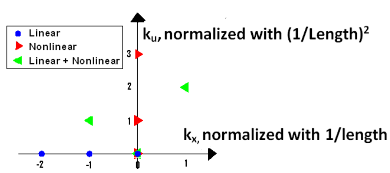

Figure 2: Optimum k-space locations are provided for only linear, only nonlinear and combination of linear and nonlinear gradients.

Methods: In order to provide better visualization of the freedom in the design, 1D inhomogeneity along the x axes is assumed and everything is assumed to be uniform in y direction in order to simplify the explanation of the method. This assumption corresponds to no B_1^+ correction along the y direction. For a slice selective excitation, spokes in the direction of k_z will be used. Due to spoke excitation, a thin slice will be excited and without loss of generality we can assume analysis will be held on z = 0 plane. Therefore, 4 dimensional functions in (2) and (3) are reduced to 2D which depends on x, u or k_x, k_u. With this assumptions, (2) reduces to summation expression in (4). Optimization problem is formulated in (4), where f(x) is the desired excitation profile, N is the number of spokes, a_n is the amplitude of the n^{th} spoke with arbitrary units, k_{xn} and k_{un} is the known location of k_x and k_u variable of the n^{th} spoke. u(x) is dependency of the NGF with respect to x. The expression in (4) can be minimized analytically for free variable a_n. Since different types of coil designs and different loadings cause various B_1^+ inhomogeneity, any type of f(x) can be necessary for B_1^+ inhomogeneity correction. Therefore, an example target excitation profile f(x) is chosen as yellow line in Fig. 1. NGF dependency is $u(x) = x^2$. Although field dependency should satisfy Laplacian equation, x^2-z^2 field dependency can satisfy Laplacian equation and at z = 0 plane, this field becomes $u(x) = x^2$. In simulations, 3 spoke optimization is implemented. Optimization problem has been solved for three different cases by using only linear gradients, only nonlinear gradients and simultaneous use of linear and nonlinear gradients. Using (4), optimization performed for a given spoke locations and the spoke locations are optimized by testing all possible combinations in a rectangular window of 7x7 spoke locations. Optimizations and simulations are performed in MATLAB (The Mathworks. Inc.,Natick,MA).

Results: Target profile is given in Fig.1. Many different combinations of spoke locations are analytically optimized for each case. In Fig.2 best spoke locations for each case is presented. After optimizing amplitudes and phases of the spokes given in Fig. 2, excitation profiles in Fig. 1 are obtained. RMSE between obtained profiles and target profiles results in 24.0%, 17.5% and 8.3% for only linear, only nonlinear and their combination correspondingly. Although SAR was not a parameter in this optimization process, normalized SAR values are calculated as 1, 1.06 and 1.03 for only linear, only nonlinear and combination of them correspondingly.

Discussion and Conclusion: In simulations, spoke locations for 3 different cases are analyzed as only linear gradient, only nonlinear gradient and combination of them. As an example target excitation profile, combined gradients produce much more accurate excitation profile. Optimizations for many different target profiles are performed and one of cases as an example is presented here. For many of the cases combination of linear and NGF produces lower RMSE. In many situations, coil geometry and loading does not have to be symmetric which makes harder to obtain the desired profile by using low spoke numbers. Therefore, target trajectory is chosen with anti-symmetric amplitude and phase distributions. However, proposed algorithm is free to converge only linear or only non-linear gradients at worst case. Furthermore, although optimization is performed for most accurate target trajectory without concerning SAR, combination of linear and NGF leads to almost equal SAR with other methods. Optimization can easily be manipulated for weighted average of homogeneity and SAR. Additionally, the proposed method is suitable to extend for multichannel RF transmitters, gradient channels and arbitrary k-space trajectories. In conclusion, nonlinear gradients and linear gradients are formulated as independent k-space variables, which can increase freedom in the trajectory design and RF pulse design in order to obtain more homogenous RF excitation.

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