Joint MR-PET reconstruction using vector valued Total Generalized Variation

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Target Audience: Researchers and clinicians interested in MR-PET imaging.

Purpose: It was recently shown that simultaneously acquired data from state-of-the-art MR-PET systems can be reconstructed simultaneously using the concept of joint sparsity, yielding benefits for both MR and PET reconstructions [1]. In this work we follow this direction and propose a new dedicated regularization functional for multi-modality imaging that exploits common structures of the MR and PET image via a coupling of their gradient and Hessian fields.

Methods: Image Reconstruction: The proposed regularization functional is based on the second order total generalized variation functional, which penalizes first and second order derivatives and has been shown to allow a high reconstruction quality in the context of MR imaging alone [2]. The central idea of the current work is that, instead of performing a TGV regularized reconstruction for each modality separately, one can regard the two images as a single multi-channel image and use a suitable extension of the TGV functional for vector valued data as regularization. In doing so, it is important to choose appropriate matrix norms for the pointwise coupling of the image gradients and Hessians. One possibility is to use the Frobenius norm which corresponds to pointwise l2 coupling of the matrix entries and hence enforces joint sparsity of the gradient and Hessian fields. Images are reconstructed by minimizing the following cost function:

$$\underset{x_{MRI},x_{PET}}{\operatorname{argmin}}\left\{\lambda\left\|E(x_{MRI})-g\right\|_2^2+\mu\sum_{j=1}^{J}\Big((A(x_{PET}))_j-f_j\log(A(x_{PET}))_j\Big)+TGV_\alpha^2\begin{pmatrix}x_{MRI}\\x_{PET}\end{pmatrix}\right\}. \quad (1)$$

In equation (1) x_{MRI} and x_{PET} are the 3D image data, *g* and *f* are MR and PET raw data. *E* and *A* are the corresponding MR and PET forward operators. λ and μ are parameters weighting the individual data fidelity terms for MR and PET. The MR operator includes coil sensitivities. Corrections for attenation, scatter and random coincidences are are included in the PET operator, as well as point spread function based resolution modelling [3]. The indices of the total number of *J* PET lines of response are denoted by *j*. TGV_α^2 is the proposed multi-channel regularizer and is defined as

$$TGV_\alpha^2(x)=\min_v \alpha_0\Big\|\left|\nabla x-v\right|_f\Big\|_1+\alpha_0\Big\|\left|\varepsilon v\right|_f\Big\|.$$

ε is the symmetrized gradient of the matrix field v, $|\varepsilon v|_f$ is the Frobenius norm of the tensor εv and $|\nabla u-v|_{\{f\}}$ is the Frobenius norm of the matrix (∇u-v). A primal-dual algorithm [4] with 500 iterations was used for numerical solution of equation (1).

Data acquisition: All in vivo data were acquired on a clinical 3T MR-PET System (Siemens Biograph mMR). This study was approved by our institutional review board (IRB), and written informed consent of all patients was obtained prior to examination. 18F-FDG was used as the PET tracer (10 mCi) and the scans were performed after an approximate uptake time of 45min. A Dixon sequence was used to generate attenuation correction maps, followed by a 10min PET acquisition. A 3D MP-RAGE sequence with the following sequence parameters was used for simultaneous MR data acquisition: TR=2300ms, TE=2.98ms, TI=900ms, FA=9°, 256 matrix, voxel size=1×1×1mm^3, 192 slices and BW=240Hz/pixel, acceleration factor 2 with 24 ACS lines at the center of k-space for calibration of coil sensitivities.

Results: A comparison of the proposed joint MR-PET TGV_α^2 reconstruction to a conventional MR inverse Fourier transform (IFT) and a PET OSEM reconstruction is shown in **Figure 1**. The nonlinear reconstruction removes the aliasing artifacts from the MR images. This is not surprising, and given the moderate amount of undersampling comparable reconstruction quality can be expected with a single modality TGV [2] or even a standard parallel imaging reconstruction. The result of the jointly reconstructed PET is more interesting and shows a noticeable reduction of noise and an improvement in the depiction of small structures.

Discussion: Our results show the benefits of a joint multi-modality image reconstruction approach, which complements the simultaneous data acquisition in combined MR-PET systems. The performance of the joint reconstruction depends upon the selection of the matrix norm that enforces the joint image properties. In this work the Frobenius norm was used, thus enforcing joint sparsity of the two modalities. A second possibility is to use the nuclear norm for the Jacobian-matrix, i.e., the l1 norm of the singular values. This enforces sparsity of the singular values and hence the gradients of the reconstructed MR and PET image are more likely to be linearly dependent. Thus common edges are preferred and aligned. The comparison of different norms is the topic of ongoing work.

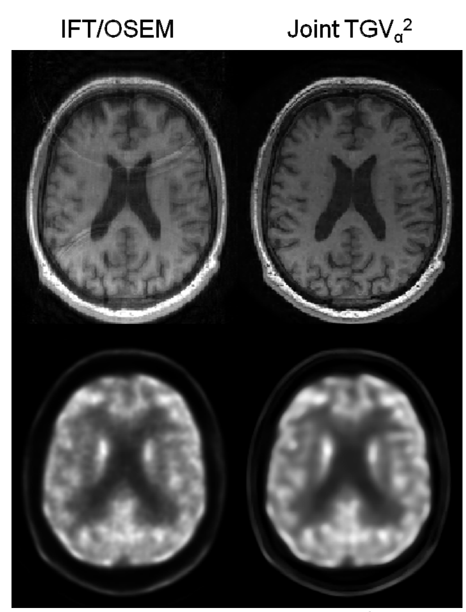

Figure 1: Inverse Fourier transform (IFT) and OSEM reconstruction (left) in comparison to the proposed joint MR-PET TGV_α^2 reconstruction. The joint PET reconstruction shows a clear reduction of noise and an improved depiction of small structures.

References: [1] Knoll et al., ISMRM 2014 p82, [2] Knoll et al., MRM 65: 480-491 (2011), [3] Koesters et al., IEEE NSS/MIC: 4365-4368 (2011), [4] Chambolle and Pock, Math Imag Vis 40: 120-145 (2010).