Highly Undersampling MR Image Reconstruction Using Tree-Structured Wavelet Sparsity and Total Generalized Variation Regularization

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Purpose: Magnetic resonance imaging (MRI) has been extensively studied for its ability to visualize the anatomical and physiologic structures as well as functional information. But MRI commonly suffers from an inherently slower data acquisition process. To overcome this problem, compressed sensing-based MRI (CS-MRI) has emerged as a promising framework for MRI reconstruction from undersampled *k*-space data [1]. Both wavelet sparsity and total variation (TV) regularization have been widely used to stabilize the reconstruction results. In conventional CS-MRI, L_1 norm of wavelet coefficients is extremely effective, but inherently there is a need to oversample at above the theoretical minimum sampling rate to guarantee exact reconstruction. Recent work has shown that non-convex optimization with L_p ($0 \le p < 1$) norm could further reduce the number of required measurements [2]. In this study, we mainly focus on L_0 norm because it can better represent the measure of wavelet sparseness. In practice, MR images not only are sparse in wavelet domain, but also tend to be tree-structured sparse [3]. Thus there is a great potential to incorporate the L_0 regularized tree-structured wavelet sparsity (TsWS) into highly undersampled MR image reconstruction. On the other hand, TV commonly tends to cause staircase-like artifacts due to its nature in favoring piecewise constant solution. To overcome the model-dependent deficiency, the second-order total generalized variation (TGV^2) regularization [4] will be introduced to suppress staircase-like artifacts while preserving tissue features.

Methods: We propose to combine L_0 regularized TsWS and TGV^2 to reconstruct MR image from highly undersampled *k*-space data. Given the undersampled Fourier transform **E** and acquired data **d**, MR image **f** can be reconstructed by considering the following minimization problem

$$\min_{\mathbf{f}} \left\{ \|\mathbf{Ef}-\mathbf{d}\|_2^2 + \lambda\left(\|\mathbf{\Psi f}\|_0 + \sum\nolimits_{s\in\Sigma} \|(\mathbf{\Psi f})_s\|_2 \right) + \eta \mathrm{TGV}^2(\mathbf{f}) \right\}, \tag{1}$$

where $\lambda, \eta > 0$ are regularization parameters, $\mathbf{\Psi}$ denotes wavelet transform, Σ represents the set of all parent-child groups and *s* is one of such groups. TGV^2 is defined as $\mathrm{TGV}^2(\mathbf{f}) = \min_{\mathbf{u}} \{ \alpha_1 \|\nabla \mathbf{f} - \mathbf{u}\|_1 + \alpha_0 \|\Im(\mathbf{u})\|_1 \}$ with symmetrized derivative being $\Im(\mathbf{u}) = (\nabla \mathbf{u} + \nabla \mathbf{u}^T)/2$. In this study, parameters α_1 and α_0 were empirically set to be 1 and 2, respectively. To achieve solution stability, we will develop an efficient alternating minimization algorithm (AMA) to solve Eq. (1). Let $\mathbf{\Psi f} = \mathbf{v}$ and $\mathbf{S\Psi f} = \mathbf{z}$, the solution of (1) is equivalent to solving the following problem

$$\min_{\mathbf{v},\mathbf{z},\mathbf{f}} \left\{ \|\mathbf{Ef}-\mathbf{d}\|_2^2 + \lambda\left(\|\mathbf{v}\|_0 + \sum\nolimits_{i=1}^{p} \|\mathbf{z}_{s_i}\|_2 \right) + \beta_1 \|\mathbf{\Psi f} - \mathbf{v}\|_2^2 + \beta_2 \|\mathbf{S\Psi f} - \mathbf{z}\|_2^2 + \eta \mathrm{TGV}^2(\mathbf{f}) \right\}, \tag{2}$$

where $\beta_1, \beta_2 > 0$ are constant parameters, **S** is a binary matrix to duplicate the overlapped entries, s_i is the *i*-th group and *p* is the total number of parent-child groups. The variables **v**, **z** and **f** are essentially separable, AMA attempts to solve Eq. (1) by decomposing Eq. (2) into three simpler subproblems. In particular, the **v**-subproblem is in essence a L_0 minimization problem, which can be solved using a hard-thresholding operator; the **z**-subproblem can be efficiently solved using a shrinkage formula; the solution of **f**-subproblem can be obtained using the fast iterative shrinkage-thresholding algorithm (FISTA) [5]. Thus we can alternatively solve Eq. (2) with respect to **v**, **z** and **f** until the solution converges to an optimal solution. The proposed MR image reconstruction method will be compared to WaTMRI [6] and WSNLTV [7]. WaTMRI was developed by combining L_1 regularized TsWS and TV; WSNLTV was proposed based on wavelet sparsity and non-local TV (NLTV) regularization.

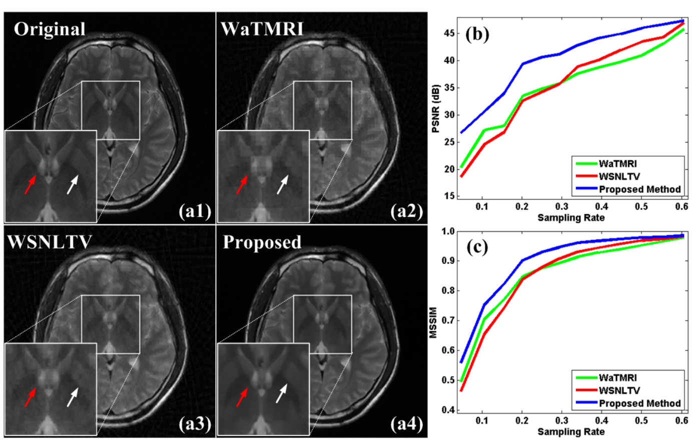

Figure 1: *In vivo* brain image reconstruction results. (a) Visual comparison of different methods for radial sampling of only 15.58% of k-space data. The enlarged regions of the images are displayed at the bottom-left corner of each image. From (b) to (c): the metrics PSNR and MSSIM are respectively used to objectively evaluate the reconstruction performance under different radial sampling rates.

Results: The reconstruction results for one *in vivo* brain dataset by WaTMRI, WSNLTV and our proposed method are presented in Fig. 1. It can be observed that our method outperforms other methods in terms of both quantitative and visual quality evaluations. As shown by the arrows, the blurred edges generated by both WaTMRI and WSNLTV result in visual quality degradation. In contrast, our proposed method is capable of preserving edges and fine structural details, which can provide a significant improvement in the visual quality. The advantage of our reconstruction method is further confirmed by the quantitative quality evaluations in terms of PSNR and MSSIM as shown in Figs. 1b and 1c. Compared with WaTMRI and WSNLTV, our method could generate a great improvement on reconstruction quality for different radial sampling rates ranged from 4.73% to 60.57%. In summary, our superior reconstruction performance benefits from the L_0 regularized tree-structured hierarchical sparsity and TGV^2 regularization.

Conclusion: In this study, we proposed to implement highly undersampled MR image reconstruction by combining both L_0 regularized TsWS and TGV^2. To achieve solution stability, the corresponding minimization problem is decomposed into several simpler subproblems. Each of these subproblems has a closed-form solution or can be efficiently solved using existing optimization algorithms. The experimental results on one *in vivo* brain dataset have demonstrated the superior performance of our proposed method.

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