Parameter-Free Sparsity Adaptive Compressive Recovery (SCoRe)

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Target Audience: Researchers interested in developing image recovery methods from undersampled MRI data.

Purpose: We develop a method that enables recovery of MR images from highly undersampled data. In particular, we propose a parameter-free recovery method, called Sparsity Adaptive Compressive Recovery (SCoRe), that models the image using a sparse representation in a union of subdictionaries, and that learns and exploits a non-uniform level of sparsity across subdictionaries. As an application example, we apply SCoRe to cardiac MRI.

Methods: MR images, like natural images, possess a rich structure that admits sparse representation under multiple, disparate transforms.[1] For this reason, redundant dictionaries are routinely used to express and exploit structure in MR images. For MRI, two common choices of redundant dictionary include: (i) the undecimated wavelet transform (UWT) and (ii) the union of a total variation transform and the discrete wavelet transform. When using a redundant dictionary, however, the level of sparsity may vary across different groups of atoms, i.e., across "subdictionaries." See Fig. 1 for an example. Therefore, using the same thresholding rule across subdictionaries may not yield acceptable results. For MRI, it is typical to formulate image reconstruction as an optimization problem of the form:

$$\underset{x}{\operatorname{argmin}}\ \|\Psi x\|_1^1 \quad \text{such that } \|y - \Phi x\|_2^2 \le \delta, \qquad (1)$$

where Φ is the measurement matrix, Ψ is a redundant dictionary, x is the image to be recovered, y is the measured data, and δ is an upper bound on the ℓ_2 norm of the measurement noise, which can be estimated from a noise-only scan. To allow for varying levels of sparsity at the subdictionary level, we propose to generalize the problem in Eq. 1 to

$$\hat{x} = \underset{x}{\operatorname{argmin}} \sum_{i=1}^{D} \lambda_i \|\Psi_i x\|_1^1 \quad \text{such that } \|y - \Phi x\|_2^2 \le \delta, \qquad (2)$$

where Ψ_i represents one of D subdictionaries, and λ_i acts as a relative weight on the i^{th} subdictionary and thus determines the extent of soft thresholding on $\Psi_i x$. From the perspective of unconstrained optimization, Eq. 2 can be viewed as a compound regularization,[2] with λ_i determining the relative strength of the i^{th} regularization term. For MRI, the values of λ_i are often selected by the user. In SCoRe, we instead estimate λ_i by iteratively maximizing the likelihood function $\mathcal{L}(\lambda_i \mid \Psi_i \hat{x})$, where $\hat{x}$ is the previous image estimate. For an i.i.d. Lapacian prior on $\Psi_i \hat{x}$, maximizing the log-likelihood function yields

$$\hat{\lambda}_i = kN \Big/ \sum_{j=1}^{N} |[\Psi_i \hat{x}]_j|, \qquad (3)$$

where $[\Psi_i \hat{x}]_j$ is the j^{th} coefficient of $\Psi_i \hat{x}$ and N is the total number of coefficients in $\Psi_i \hat{x}$. Here, $k = 1$ for real $\Psi_i \hat{x}$, and $k = 2$ for complex $\Psi_i \hat{x}$. The implementation of SCoRe includes inner iterations to solve Eq. 2 using Douglas-Rachford splitting[3] and outer iterations (Eq. 3) to update λ_i. For the data presented in this work, we chose the number of outer and inner iterations to be 16 and 12, respectively. ***Simulation***: We evaluated SCoRe using a 120×120 dynamic digital phantom with 48 temporal frames and 12 simulated channels. The data were undersampled at $R = 4$, 8, 12, and 15 using VISTA sampling.[4] ***Experiment***: Real-time, free-breathing cardiac cine data were acquired with VISTA (3T Siemens, 32 channel cardiac array) at four different acceleration rates ($R = 4$, 8, 12, and 15) from two healthy volunteers. Other imaging parameters included: 48 frames, 224×144 matrix size, 360×288 mm^2 FOV, SSFP sequence, 1 ms TE, and 2.7 ms TR. In our SENSE-based implementation of SCoRe, the coil sensitivities were estimated from fully sampled, time-averaged data.[5] Here, we used 3D UWT of the spatiotemporal image for Ψ. For SCoRe, the subband weights (λ_i) were iteratively estimated using Eq. 3. For comparison, a more typical, equal-weight (EW) approach, where all eight subbands of UWT were treated identically (Eq. 1), is also presented.

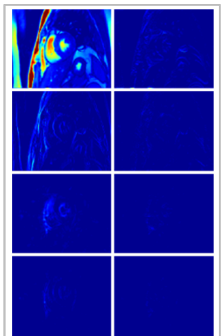

Fig. 1: A frame from each of the eight subbands of 3D undecimated Haar wavelet transform of a cardiac dataset. The varying level of sparsity across subbands is evident.

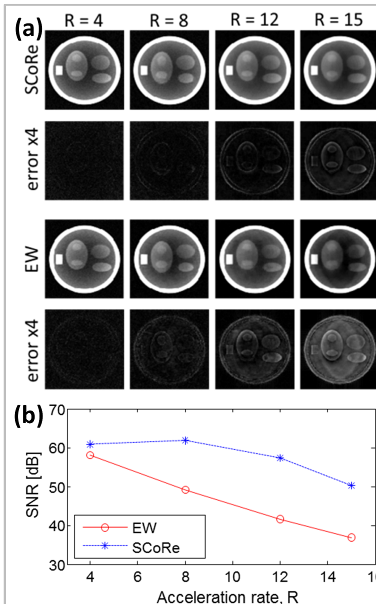

Fig. 2: Simulation results. (a) Representative frames from SCoRe as well as from a more typical EW approach (Eq. 1). (b) Recovery SNR as function of acceleration rate.

Results: Fig. 2 shows the phantom results from one of the 48 frames. SCoRe generated images with higher recovery SNR ($-20\log(\|x - \hat{x}\|/\|x\|)$) and lower visible artifacts. The difference between the EW approach and SCoRe was more pronounced at higher acceleration rates. At rate 15, SCoRe had 15 dB higher recovery SNR. Fig. 3 shows results from real-time, free-breathing MRI at rate 12. As evident from the representative frames, SCoRe suppressed aliasing artifacts that are visible in the EW approach. For SCoRe, the computation time for the phantom and in vivo datasets was approximately 3 to 5 minutes per dataset.

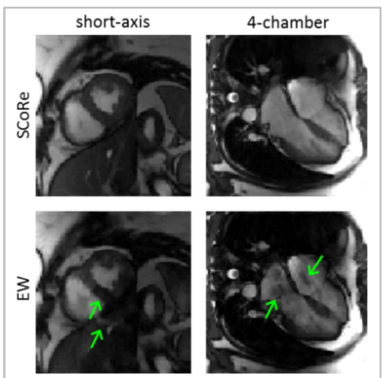

Fig. 3: Representative frames from volunteer imaging at R=12. The arrows highlight some of the visible artifacts for the EW approach.

Discussion: While redundant dictionaries are often employed to exploit sparsity, different dictionary components (i.e., subdictionaries) may not yield representations with the same levels of sparsity. For example, for most MR applications, the wavelet approximation coefficients are rarely sparse. Likewise, in dynamic MR applications, the temporal dimension may exhibit more sparsity than the spatial dimensions. Since subdictionary sparsity levels are not known in advance, the user is left to either treat all subdictionaries identically or manually tune the regularization strengths for each subdictionary, which can easily become intractable when there are multiple subdictionaries involved. Here, we have proposed a method that adapts, in a data-driven manner, to the inherent level of sparsity in each subdictionary. Moreover, the proposed adaptation does not require the use of any additional tuning parameters. In fact, SCoRe uses no tuning parameters beyond the measurement noise power, which, for MRI, can be easily estimated in advance.

Conclusions: We have presented a new data processing method that adapts to the sparsity of each subdictionary. The proposed approach can be applied to a variety of MRI applications and potentially eliminate the need to manually tune the weights for the subdictionaries. Dynamic MRI applications, in particular, can benefit from SCoRe due the inherent differences in the sparsity of spatial and temporal dimensions.

References: [1] Candes EJ et al., ACHA, 31(1) 59-73, 2011. [2] Afonso MV et al., IEEE ICIP, 4169-4172, 2010. [3] Combettes et al., IEEE J-STSP, 1(4) 564–574, 2007. [4] Ahmad et al., to appear in MRM, DOI: 10.1002/mrm.25507. [5] Walsh DO et al., MRM, 43(5) 682-690, 2000.