MRI Constrained Reconstruction without Tuning Parameters Using ADMM and Morozov's Discrepency Principle

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PURPOSE: Constrained reconstruction algorithms generally rely on manual selection of one or more tuning parameters in order to deliver good image quality. This non-trivial task has motivated a board range of automated parameter selection methods based on quantitative measures [1-7]. Methods based on Stein's unbiased risk estimate (SURE) have shown promise for MRI constrained reconstruction [4]. However, for ill-posed inverse problems, it is difficult to estimate MSE using SURE since the observed data contains only partial information about the true image; also, analytically evaluating SURE is difficult for iterative methods since it requires exactly differentiating the operations in nearly every computation step. To overcome these two disadvantages, Weller et al. [5] proposed a Monte-Carlo SURE framework based on regularized auto-calibrating parallel MRI reconstruction. They demonstrated that a wide range of parameter values would produce similar results, indicating precise estimation of MSE can be compromised. Given this, we propose an algorithm that does not require differentiating the computation operations, and is straightforward to implement. We show that this framework iteratively determines the tuning parameter based on the data consistency term and provide similar reconstruction result with manually selected parameters.

THEORY: Constrained reconstruction for under-sampled k-space data can be formulated as a minimization problem:

$$\hat{\underline{m}} = \arg\min \left\| F_u S \underline{m} - \underline{k} \right\|_2^2 + \lambda \left\| \Psi \underline{m} \right\|_1 \qquad \textbf{(Eqn. 1)}$$

or equivalently:

$$\hat{\underline{m}} = \arg\min \left\| \Psi \underline{m} \right\|_1, s.t. \left\| F_u S \underline{m} - \underline{k} \right\|_2 \le \varepsilon \qquad \textbf{(Eqn. 2)}$$

Since ε is in general less difficult to estimate than λ, we propose an algorithm that directly solves **Eqn. 2**. Chambolle [6] showed that the error term $\left\| F_u S \underline{m} - \underline{k} \right\|$ is a non-decreasing function, denoted as $f(\lambda)$, of the shrinkage parameter λ, and that to make $f(\lambda)$ converge to the estimated value of ε, an effective iterative adjustment is $\lambda^{(n+1)} = \lambda^{(n)} \left(\varepsilon / \left\| F_u S \underline{m} - \underline{k} \right\| \right)$ [6-7]. According to Morozov's discrepancy principle [8], ε can be estimated as the number of samples times the noise standard deviation per sample, measured with a RF-off scan or estimated by robust median absolute deviation [7]. λ is automatically adjusted during iterations so the correction factor $\varepsilon / \left\| F_u S \underline{m} - \underline{k} \right\|$ will converge to 1.

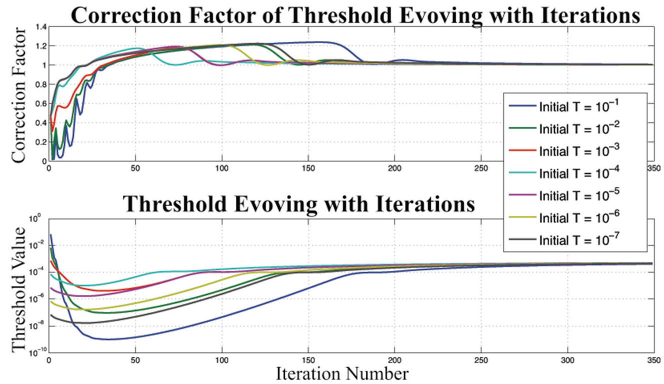

Figure 1 Shrinkage parameter evolves during iterations

METHODS: Raw k-space data were collected from 7 patients scheduled for routine brain MRI on a 3T Signa EXCITE HDxt system with an 8-channel head coil. Raw data from two sequences were used: (1) 512×346×28 2D multi-slice oblique-coronal T2 weighted spin echo (T2-SE) with TE/TR of 102/6367 milliseconds, echo train length 15; (2) 320×192×28 2D multi-slice oblique-coronal T2 weighted fluid-attenuated inversion recovery (T2-FLAIR) with TE/TI/TR of 150/2200/8900 milliseconds. The raw k-space data was retrospectively under-sampled. The under-sampling mask and tuning parameter for comparison was selected by experienced radiologist, as described in Ref [9]. We implemented the reconstruction using Alternating Direction Methods of Multipliers (ADMM) framework [10], and updated the shrinkage parameter λ in each iteration step.

Accel Rate	Manual ($\times 10^{-3}$)	Automatic ($\times 10^{-3}$)
T2-SE		
2x	2.0	1.8±0.8
3x	2.0	1.5±0.9
4x	2.0	1.4±0.9
T2-FLAIR		
2x	2.0	2.0±0.8
3x	1.0	1.8±0.6
4x	1.0	1.0±0.2

Table 1. Comparison of manually selected and automatically determined tuning parameters. The proposed method was applied to 7 data sets, and parameter is shown as mean±std dev.

RESULTS AND DISCUSSION: This adaptive framework required 3 times as many iterations to learn the parameter λ. As shown in **Fig.1**, the threshold value eventually converged to the same value for each data set with different initial guess across several orders of magnitude. We observed that underestimation of the noise level ε would lead to a correction factor always smaller than 1 and a trivial value of the threshold λ. As a result, a good estimation of noise level is crucial to this algorithm. The converged parameter selection results by proposed method are shown in the **Table. 1**. The resulting reconstruction quality is comparable to the use of tuning parameters manually selected by Radiologists. Representative images are shown in the **Fig. 2**.

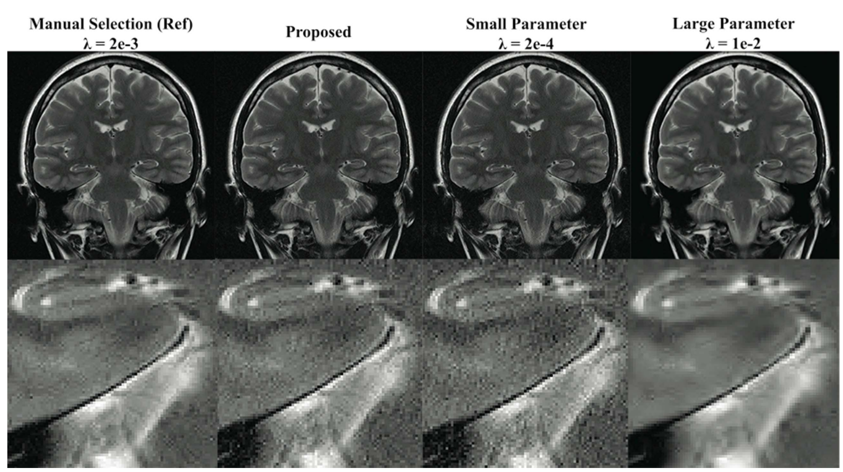

Figure 2 Example result: T2w-SE with rate-3 undersampling and constrained reconstruction. (bottom) zoomed view of the hippocampus that illustrates the depiction of fine structures.

CONCLUSION: Tuning parameters can be determined iteratively, eliminating the need for manual parameter selection, but requiring extra iterations. Our preliminary results indicate that this approach provides image quality comparable to manual parameter selection. Future work will extend this algorithm to multiple constraint terms, and investigate experimental validation using various kinds of data sets.

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