Simultaneuos Magnitude and Phase Regularization in MR Compressed Sensing using Multi-frame FREBAS Transform

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Introduction : Recent compressive sensing (CS) application to MRI results show that it is possible to reconstruct MR images from relatively few linear measurements, however, its use in applications with rapid spatial phase variations is difficult, since not only the magnitude but also phase regularization is required in the CS framework. An iterative MRI reconstruction with separate magnitude and phase regularization was proposed for applications where magnitude and phase maps are both of interest [1]. Since this method requires the approximation of phase regularizer to cope with phase unwrapping problem, it is roughly 10 times slower than conventional CS and the convergence is not always guaranteed. We have proposed a faster reconstruction algorithm in which real- and imaginary-part of complex image are reconstructed independently using the symmetrical signal trajectory [2]. In this article we propose a novel image reconstruction scheme for CS-MRI in which phase regularizer or symmetrical sampling trajectory are not required in the rather simple CS reconstruction scheme, but highly robust to rapid phase changes.

Theory: We use a kind of multi-resolution image decomposition analysis based on the Fresnel transform (FREBAS: FREsnel Band Split transform) [3] for sparsifing function. FREBAS transform is made up of two different algorithm of Fresnel transform which allows optional scaling of images, i.e. FREBAS scaling parameter *D* can take the value not only the integer but also real number. Figure 1 shows the examples of FREBAS transformed signal and corresponding complex-value basis. These transforms behave differently for smooth parts and textures. For oriented textures, FREBAS with smaller scale *D* lead to a significantly sparser expansion than larger scale *D*, on the contrary, FREBAS with larger scale *D* lead to a sparser expansion than smaller scale *D* for rapid changes of magnitude or phase. Therefore, we propose to solve an optimization problem that involves multi-frame transforms of the image based on the FREBAS.

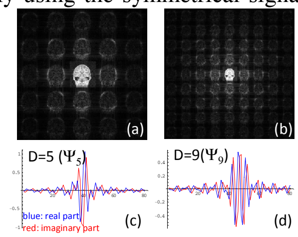

Fig.1 Examples of FREBAS transform and its basis functions

Let us consider the compressed sensing problem with incomplete measurements *f* with measurement matrix Φ and MR image *u*. We consider the generalized optimization problem using the multi-frame transform matrices $\Psi_{D[m]}$ which uses scaling factor *D*[*m*] of FREBAS transform.

$$\operatorname{argmin}\left\{ J(u) + \frac{1}{2}\left\| f - \Phi u \right\|_2^2 \right\} \qquad \cdots (1) \qquad\qquad J(u) = \sum_{m=1}^{n} \left\| \Psi_{D[m]} u \right\|_1 \qquad \cdots (2)$$

Algorithm 1.

Input : $\beta = 1/L,\ \ u^0 = r^1$
for $k = 1$ to K do
  $u_1 = r^k$
  for $m = 1$ to n do
    $x_m = \Psi_{D[m]} u_m \qquad (m = 1,2,\ldots n)$
    $\xi_m = x_m - \beta_k \nabla(x_m)$
    $x_{m+1} = prox(\|\xi\|_1)(\xi_m)$
    $u_{m+1} = \Psi^{-1}_{D[m]} x_{m+1}$
  endfor
  $r^{k+1} = u_n$
  $\beta_{k+1} = \eta \beta_k$
endfor

We used the SpaRSA based reconstruction method shown in Algorithm 1 to solve the convex optimization problem. Figure 2 shows an example of aliasing artifact reduction by applying multi-frame FREBAS transform. The PSF for a point image after random sampling in the phase-encoding direction is shown in Fig. 4(a) for a signal reduction factor of 30%. The PSFs after FREBAS domain (*D*=4) thresholding (threshold level: 0.04) are shown in Fig. 4(b). Figure 4(c) and (d) shows the result of 4-step thresholding using *D*=4 in all step and *D*[*m*] increasing from 4 to 7, respectively. Artifacts are significantly reduced in (d) compared to (c). This indicates that mutual incoherence between basis of multi-frame FREBAS transform and measurement matrix is smaller than that of single-frame FREBAS and Fourier operator. This contributes to superior image reconstruction by the proposed CS method.

Results and Discussion: MR normal volunteer images were collected using a Toshiba 1.5T MRI scanner. Flow-sensitive black blood images were acquired in order to obtain images that have locally strong phase distortions due to blood flow (TE/TR = 40/50 ms, 256×256 matrix, slice thickness: 1.5 mm × 50 slices). The signal for the phase encoding direction, except for the central region (20-lines), is randomly selected in order to simulate a given reduction factor. Reconstruction was performed using an Algorithm 1 for 40 images without the use of total variation to evaluate the performances of sparsifying function itself. The scale parameters of FREBAS *D*[*m*] used in the multi-frame reconstruction were determined by preliminary experiments, i.e. *D*[*m*] increases from 3 to 11 with 1 incremental step. The execution time for 90 iterations is 8.4s for single image reconstruction using 3.5-GHz Intel Core i7-3770 processor (gnu c compiler). Figure 3 shows the averaged peak-signal-to-noise ratio (PSNR) and RMS of phase error as a parameter of signal reduction factor of proposed method and CS with symmetrical trajectory (Sym-CS) [2]. The PSNR was improved in the proposed method. Figure 4 shows the comparison of reconstructed images among proposed method, Sym-CS and single-frame FREBAS (*D*=7) using 40% signal. The magnitude of images in the proposed method capture the object in far greater detail and with smaller aliasing artifacts. Phase profiles on the red dashed line in Figs. (e)-(g) are shown in (i) to (k). Proposed method can restore the phase on the image much better than the Sym-CS, especially at the region pointed by red and blue arrows.

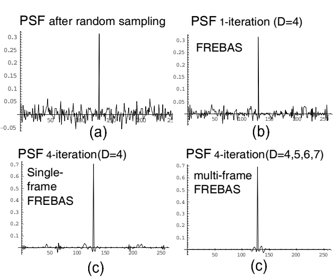

Fig.2 Aliasing artifact reduction after multi-frame FREBAS domain thresholding

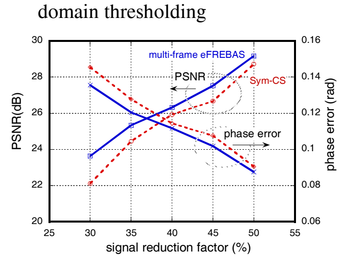

Fig.3 Comparisons of PSNR and phase error between proposed method and CS using symmetrical signal trajectory (Sym-CS).

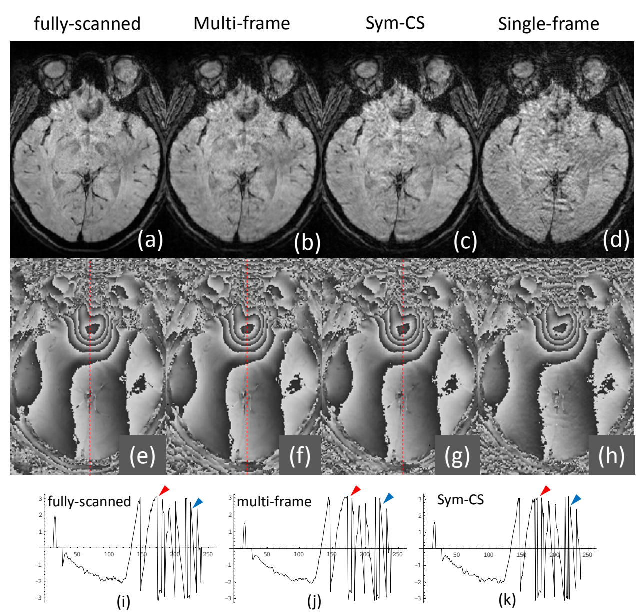

Fig.4 Comparison of reconstructed images using 40% signal. Proposed method (b) is most close to fully-scanned image in the sense of amplitude and phase.

Conclusion: A new sparsifying function for phase-varied images in CS-MRI which is simple and robust to phase distortions is proposed. Because the proposed method do not require phase regularizer, a faster and precise reconstruction will be achieved.

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