

Overcoming the Image Position-Dependent Resolution Inherent in DFT and CS Reconstructions

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Target Audience: Those interested in new approaches to obtain the highest possible MR resolution during discrete Fourier transform (DFT) reconstruction of fully sampled and compressed sensing (CS) reconstruction of under-sampled truncated k -space data sets.

Purpose: In magnetic resonance imaging (MRI), there is always a compromised balance between high spatial and temporal resolutions. Truncated k -space data sets provide higher temporal resolution but compromise spatial resolution by introducing truncation artifacts and other distortions during DFT reconstruction [1]. CS reconstruction [2], using under-sampled data, is used to improve spatial resolution while retaining temporal resolution. It has been suggested that certain Fourier domain properties can produce MRI reconstructions with resolutions that are dependent on the position of an object in the final reconstructed image [3]. We demonstrate this position dependent resolution and propose two approaches to overcome it.

Methods: A high-resolution magnetic resonance (MR) image (A) was obtained through the DFT reconstruction of the 512×512 k -space data of a GE phantom. Lower resolution images were generated by truncating the 512×512 k -space data set to 128×128 and 108×108 followed by DFT and CS reconstructions. The CS reconstructions via the discrete cosine transform (DCT) were performed using the parameters specified in [2]; 33% random under-sampling around a 10% fully sampled core. The 108×108 truncation size was empirically determined to be the greatest truncation at which the tines of the GE phantom comb (lower centre in A) remained partially resolvable following DFT reconstruction. Low resolution images of the GE phantom shifted in position by up to 0.3 cm, 1% of the field of view (FOV), were generated using the Fourier relationship that a x -direction spatial shift of $p\%$ of the FOV is equivalent to multiplying the k -space data by $\exp(-j2\pi pk/N)$ for a $N \times N$ image.

Results: The GE phantom comb tines are clearly resolvable (A) for the 512×512 DFT reconstruction. Comparison of the DFT reconstruction of the 128×128 truncated data sets of the (B) 0.54% and (C) 0.10% FOV-shifted phantoms shows different parts of the comb tines having increased resolution; evidence of a position dependent resolution inherent in DFT reconstruction as suggested in [1, 3]. The low resolution area for the DCT CS reconstructions shifts from (D) the 1st and 3rd tines, 0.54% FOV shift, (E) to the 2nd and 4th tines, 0.34% FOV shift to (F) the 3rd and 5th tines for 0.10% FOV shift.

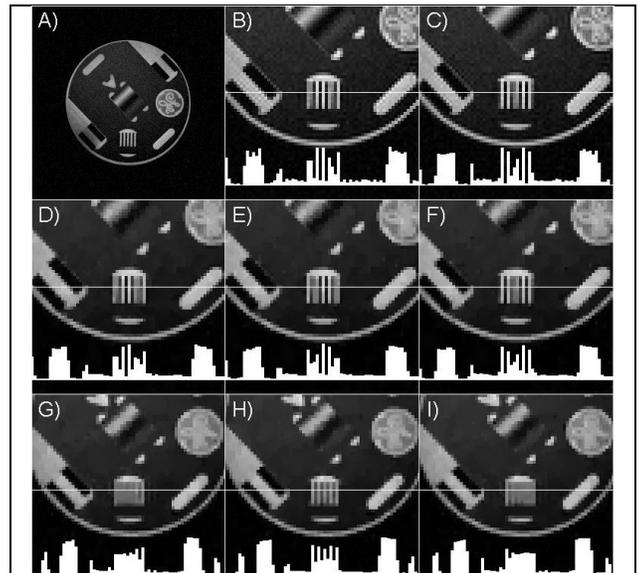
Discussion: Results (B) – (F) demonstrate the existence of position dependent MR resolution [3] observable for high resolution objects reconstructed from truncated k -space data sets using DFT and DCT CS reconstructions. The changes in resolution can be interpreted through an understanding of the successes and limitations of applying the DFT. Following the use of DFT, certain sinusoids in one domain will appear as spikes in the other domain. Other slightly higher or lower frequency sinusoids will appear smeared in the other Fourier domain through a phenomenon defined as spectral leakage [1]. Harris' time-domain observation [1] can be re-interpreted in an MR context as follows [3]. Certain k -space components, but not all, become 'natural basis' functions when the infinite continuous k -space data set is finitely sampled and truncated to $N \times N$ leading to image components with higher resolution than those generated from non-basis function components.

The position dependant changes present in the 108×108 DCT CS reconstructions provide further evidence that certain k -space components are forced to become close to basis functions with a narrow point spread function in the other Fourier domain, appearing as a sharper image components. The resolution changes in images (G) 0.04% FOV shift and (I) 0.83% FOV shift respectively show the 4th and 5th tines and the 1st and 2nd tines are closer to being represented by k -space basis function than the other tines. In (H) the truncation from 128×128 to 108×108 combined with a 0.44% FOV shift leads to all 5 tines being resolved after k -space representations have been manipulated to become basis functions.

The same images indicating a limitation of DFT and CS reconstructions can also be re-interpreted as suggesting two approaches to improve the resolution of narrow objects at specific positions with the FOV. The first preprocesses the truncated data set via the Fourier Shift operator to use k -space manipulations to move the object of interest out of a low resolution position in the image (not represented as a k -space basis function) to an area of higher resolution (object represented by a k -space basis function). The second combines Fourier Shift preprocessing with an area specific additional truncation. It is practical to custom design the 'best' k -space truncation to digitally enhance the resolution of an area if the anticipated resolution associated with a specific highly detailed object can be determined. The appropriate Fourier shift operator can then be used to manipulate (modulate) a greater region of the imaged object to acquire the characteristics of k -space basis functions giving a region of maximum resolution.

Conclusion: We have demonstrated that position dependent resolution is inherent in the DFT and CS reconstructions of truncated MR k -space data sets. This positional dependent of resolution adversely affects the interpretation of the characteristics of narrow objects. Two preprocessing approaches, Fourier Shift image preprocessing and Area Specific Additional Truncation, were shown to digitally enhance feature resolution.

References [1] F. Harris. "On the use of windows for harmonic analysis with the discrete Fourier transform", *Proc. I.E.E.E.* 66, 51 – 53. 1978, [2] M. Lustig et al., "Sparse MRI: the application of compressed sensing for rapid MR imaging", *MRM*, 58#6, 1182-1195, 2007, [3] M. Smith et al., "Impact of DFT Properties on the Inherent Resolution of Compressed Sensing Reconstructed Images", in *Proc. Irish Signals and System, Letterkenny* 2013.



A) Original 512×512 DFT reconstruction. 128×128 DFT reconstruction for B) 0.54% and C) 0.10% FOV shifts. 128×128 DCT CS reconstructions for D) 0.54%, E) 0.34% and F) 0.10% FOV shifts. 108×108 DCT CS reconstructions for G) 0.04%, H) 0.44% and I) 0.83% FOV shifts. The bar plots are the intensities of the cross-section through the GE phantom tines marked by the thin white line.