

An Approach to Improve the Effectiveness of Wavelet and Contourlet Compressed Sensing Reconstruction

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Target audience: Those interested in exploring the effectiveness of new compressive sensing (CS) approaches to improve the spatial resolution of textures present in MR reconstruction of truncated k -space data sets to allow the discrimination of neurological diseases.

Purpose: Reconstruction of MR images with high spatial resolution is a prerequisite for the application of texture analysis of MR brain images for discrimination of neurological diseases. CS reconstruction techniques achieve faster throughput (higher temporal resolution) by under-sampling truncated k -space data while retaining high spatial resolution through the application of sparseness properties [1]. The explanations for distortions introduced during discrete Fourier transform (DFT) reconstruction of truncated k -space data sets are well established: e.g. truncation artifacts are introduced by discontinuities across the ends of the k -space data set which mathematically must be considered as a cyclic data set during DFT reconstruction [2]. Recently it has been suggested that these same k -space continuity constraints must also impact the achievable resolution during CS reconstructions which repeatedly uses the DFT and $IDFT$ to move data between k -space, image and other domains [3]. A high resolution k -space extrapolation enabled (*Hi-KEE*) CS variant was proposed to overcome the k -space discontinuity issue [3]. In this paper we qualitatively compare the image texture characteristics of a MR brain data set following DFT , CS -Wavelet [1] and CS -Contourlet [4] reconstructions with new *Hi-KEE* variants of these approaches, *Hi-KEE-CS-Wavelet* and *Hi-KEE-CS-Contourlet*.

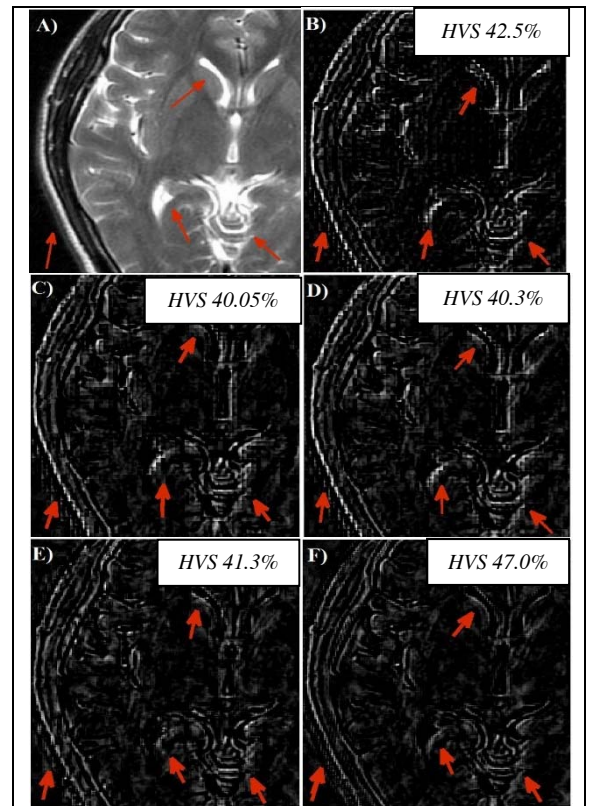
Methods: An under-sampled CS data set was prepared by truncating a high-resolution 512×512 k -space data set [5] to 128×128 followed by a random variable density 33% sampling pattern including a fully sampled 10% core of size 0.1 as per [1]. The software setting for CS -Wavelet [1] and CS -Contourlet [4] were left unchanged. As suggested in [3], the *Hi-KEE* data sets were prepared by zero padding both the CS data set and the CS sampling pattern to 256×256 before applying the CS -Wavelet and CS -Contourlet algorithms. The High Dynamic Range Visual Difference predictor (HDR - VDP -2.0) [6], a measurement criteria based on the human visual system (HVS) was used to compare CS and *Hi-KEE* images with the reference (original 512×512 data set) to determine whether a human observer would likely notice a difference between the reconstructions. The HDR - VDP -2.0 toolbox provides a visual representation, a difference probability map, and a HVS -relevant metric, a numerical value indicating the quality of image.

Results: The arrows indicate regions of interest in the images. Image (A) shows the original high resolution 512×512 brain dataset. The higher VDP intensities in the probability maps for (B) 128×128 $IDFT$ and (C) 128×128 CS -Wavelet reconstruction show a higher probability of a difference being detected by a human observer. The probability difference map (D) for 128×128 CS -Contourlet shows lower intensity over the same areas marked with red arrows, indicating the better recovery of contours. The same components are even more clearly recovered using (E) 256×256 *Hi-KEE-CS-Wavelet* of the 128×128 truncated data set. The least intensive image of (F) 256×256 difference probability map for *Hi-KEE-CS-Contourlet* represents the smallest probability of difference detection by a human observer

Discussion: The lower intensity in the quantitative probability difference maps indicate the CS -Contourlet outperforms the CS -Wavelet sparse transforms by providing a sparser representation of smooth contours with much less complexity and optimal approximation rate. The VDP - HVS relevant quality metric indicated the superiority of *Hi-KEE-CS-Contourlet* with resolution enhancement compared to the CS -Wavelet, *Hi-KEE-CS-Wavelet* and $IDFT$ reconstructions

Conclusion: An investigation of the theoretical impact that CS reconstruction has on the characteristics of k -space data has led to insight into the limitation of the CS -Wavelet reconstruction approach and led to a more reliable reconstruction algorithm which makes use of k -space extrapolation properties inherent in CS reconstruction.

References: [1] M. Lustig et al., "Sparse MRI: the application of compressed sensing for rapid MR imaging", *MRM*, 58(6), 1182-95, 2007; [2] F. J. Harris. "On the use of windows for harmonic analysis with the discrete Fourier transform", *Proc. I.E.E.E.* 66, 51-3, 1978; [3] M. R. Smith, et al., "Impact of DFT Properties on the Inherent Resolution of Compressed Sensing Reconstruction Images", in *Proc. Irish Signals and System 2013*. [4] X. Qu, et al., "Iterative thresholding compressed sensing MRI based on contourlet transform", *IPSE*, 18(6), 2010. [5] (http://www.quxiaobo.org/software/software_Contourlet_CS_MRI.html); [6] R. Mantiuk, et al., "HDR-VDP-2: a calibrated visual metric for visibility and quality predictions in all luminance conditions.", *ACM TOG*, 30(4), 2011.



A) Original 512×512 $IDFT$ reconstruction. HDR - VDP visualization maps of the probability of a human observer being able to distinguish between B) 128×128 $IDFT$ reconstruction, C) 128×128 CS -Wavelet, D) 128×128 CS -Contourlet, E) 256×256 *Hi-KEE-CS-Wavelet* and F) 256×256 *Hi-KEE-CS-Contourlet*