Undersampled High-frequency Diffusion Signal Recovery Using Model-free Multi-scale Dictionary Learning

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Target Audience: MR researchers and clinical scientists working on Diffusion MRI (diffusion signal under-sampling, recovery, intra-voxel crossing fiber, etc.)

Introduction: Low Signal-to-Noise Ratio (SNR), especially at high b-values, is a critical problem for Diffusion MRI (dMRI). Methods with different signal models [1-3] may fail to reconstruct under-sampled data from noisy measurement. Diffusion MRI signal contains redundancy as a multi-dimensional signal in both k-space and q-space. Here we proposed a novel approach to recover signal without explicitly enforcing any physical signal models. The method is model-free but learns the multi-dimensional redundancy, including the redundancy between neighborhood voxels, different directions and samples with low and high b-values, from training samples. A Dictionary Learning approach is used to recover under-sampled high-frequency (high b-value) signals in q-space. Quantitative results demonstrate the method can more accurately predict high b-value signal (>3000s/mm^2) from low b-value signal. Also it produces more accurate physiological metrics such as Generalized Fractional Anisotropy (GFA) and Orientation Distribution Function (ODF) that potentially help to resolve intra-voxel crossing fibers.

Methods: Diffusion signal contains redundancy in high-dimensional information. Dictionary Learning has been proposed to exploit the redundancy and enforce the sparsity for Diffusion MRI de-noising [4]. Here we extended idea to dMRI signal recovery by learning a multi-scale dictionary based on the observation:

i) Signals acquired with low and high b-values are correlated. Conventionally exponential or bi-exponential based physical models have been proposed. Here a dual-dictionary model was used and two dictionary $\boldsymbol{D^L}$ and $\boldsymbol{D^H}$ was trained by optimizing summation of data consistency term (with low b-value samples x^L and high b-value samples x^H) and L_1 regularization (on sparse representation coefficient α): $\min_{D_L,D_H,\alpha} \sum_{i=1}^{n} \|x_i^L - D^L\alpha\|^2 + \|x_i^H - D^H\alpha\|^2 + \lambda\|\alpha\|_1$

ii) There is redundancy in signals with adjacent angular directions since it reflects the fibers within voxels. iii) The natural image properties in dMRI results in the redundancy in non-local image patches. Thus, we consider neighbor voxel into the dictionary. We extended the dual dictionary $\boldsymbol{D}$ to model multi-dimensional information. The entire steps of the proposed diffusion signal reconstruction method are: 1) Get Training diffusion data (50% voxels are used with cross-validation). 2) Stack data for each voxel as a 4D sample (2D for 3x3 nonlocal patch, 1D for angular and 1D for b-values) and reshape into a feature vector 3) Train the high dimensional dictionary based on the features extracted from training data. 4) Reconstruct under-sampled data using the trained high-dimensional dictionary by solving:

$$x_{opt} = [D^L D^H]\alpha_{opt}, s.t.\ \alpha_{opt} = \arg\min \sum_{i=1}^{n} \|x_i^L - D^L\alpha\|^2 + \|x_i^H - D^H\alpha\|^2 + \lambda\|\alpha\|_1$$

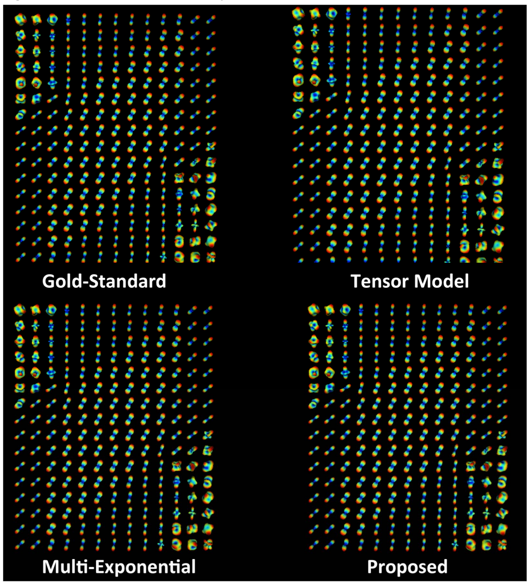

Figure 1. The reconstructed ODF using the recovered under-sampled high-frequency (high b-value) q-space signals. The proposed method results in closer metrics to the golden standard with lower error in both signal recovery and metrics estimation.

Data: Here the proposed methods were implemented with the q-space data from MICCAI 2014 *Sparse Reconstruction Challenge for Diffusion MRI* (SPARC dMRI). We used the released gold-standard data based on extensive acquisition on adiffusion phantom [5]. The data set has spatial dimension of 13x16x1 voxels and there are 5 b-values {1000,2000,3000,4000,5000} on 81 gradient directions on each shell for each voxel. The released Gold Standard data was used as ground-truth. Diffusion signal at b-value of 4000 and 5000 are under-sampled in simulation and reconstructed using various methods based on samples with low b-values (1000,2000,3000). The proposed method was compared with Tensor Model based method [1] as well as multi-exponential based method [2]. Normalized-Mean-Square-Error (NMSE) of estimated signal was computed to evaluate signal recovery accuracy. Generalized Fractional Anisotropy (GFA) and Orientation Distribution Function (ODF) was reconstructed using CSA-ODF routine in *dipy* [6]. Root-Mean-Square-Error (RMSE) of ODF and GFA was evaluated to address the performance of resolving detail physiological metrics.

Results: Table 1 lists the signal reconstruction performance of different methods. The proposed method has superior performance in recovering high b-value signals within 1% NMSE from the ground-truth. In comparison, the results released in the workshop showed the NMSE from 4%~35% and most of the methods achieving around 20% [5]. Besides, compared with conventional method, the proposed method did better job in reconstructing the physiological metrics. The produced ODF and GFA maps using the proposed method are closer to the ground truth (4% RMSE). Detail results are shown in Table 1 and Figure 1-2.

Discussion: The benefits of better signal recovering shown in figure prove that the proposed method can result in more accurate signal and metrics reconstruction. The method recovers the under-sampled high frequency (high b-value) signals in q-space without enforcing models but based on real diffusion data. This method can help to improve spatial-angular resolution and potentially resolve the intra-voxel crossing fiber issues. The method can also be applied to common dMRI pipelines for de-noising and Compressed Sensing reconstruction of under-sampled data in DWI, Tractography estimation as well as DSI.

References: [1] Inglis 2001 [2] Le Bihan 2003 [3] Zhang 2012 [4] St-Jean 2014 [5] http://projects.iq.harvard.edu/sparcdmri [6] Garyfallidis 2014

Comparison of Estimation Accuracy

Comparison	B-value.	Tensor Model	Multi-Exp.	Proposed
Signal NMSE	4000	11.5%	4.2%	0.6%
Signal NMSE	5000	21.9%	4.4%	0.7%
GFA RMSE(%)		7.2%	4.8%	4.0%

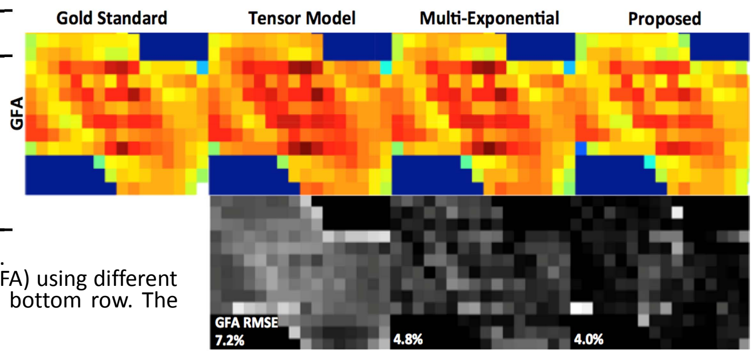

Table 1. Detail estimation accuracy of different methods (above).
Figure 2. Estimation of the Generalized Fractional Anisotropy (GFA) using different methods are shown in top row. The error maps are shown in bottom row. The proposed method achieves the best estimation performance.