A Bayesian Approach to the Partial Volume Problem in Magnetic Resonance Fingerprinting

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Target Audience: Scientists interested in the partial volume effect in MRI.

Purpose: Magnetic Resonance Fingerprinting (MRF) [1] is a technique to produce quantitative maps of tissue parameters such as T_1 and T_2 relaxation times by matching the acquired MR signal to a predefined dictionary of signal evolutions. One issue with this technique is that voxels exhibiting the partial volume effect are assigned only one dictionary entry. Recent work [2] in this area has shown that satisfactory fraction estimates can be computed per voxel using the pseudoinverse of a subdictionary containing predefined tissue types. In this work, we propose a method of solving the partial volume problem without defining in advance which tissue types the mixed signal might be composed of, rather we use an iterative process to prune the dictionary down to a cluster of representative tissue types. Though going down this path increases the mathematical and computational complexity of the problem, it is a first step toward solving the general partial volume problem encountered in every MRI scan – in a non-biased way. A statistical Bayesian framework [3] is employed to model the problem using the maximum *a posteriori* estimator as a solution. In the Bayesian approach, we model not only the likelihood distribution, but we also encode any *a priori* beliefs we have about the unknowns into the prior probability distribution. For this application, the unknowns of interest are the T_1, T_2 combinations that are present in the mixed signal and their respective ratios.

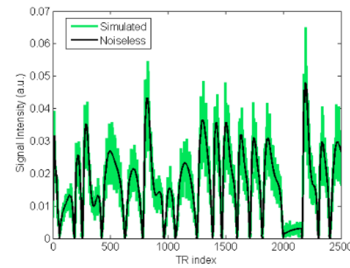

Figure 1: Simulated mixed signal evolution. The black signal is the true signal and the green represents effects from aliasing and noise.

T_1	T_2	Fraction
800	60	0.44
1700	180	0.56

Table 1: True components of the mixed signal.

Methods: The MRF dictionary is a collection of *n* simulated signal evolutions denoted $d_j,\ j=1,\ldots,n,$ where each entry is formed using a unique pair of T_1 and T_2 relaxation times. Denote by *y* an observed MRF signal evolution modeled as a weighted sum of the dictionary, $y=\sum_{j=1}^{n}\alpha_j d_j+\xi$, for weights α_j, $\sum_j \alpha_j = 1$. We assume for the time being that the term ξ accounts for both the aliasing and noise present in acquired MRF signals. Since we expect that *y* is actually made up of only a few (two or three) component signals from the dictionary, we encode the sparsity of the weights in terms of a gamma distribution, as the expected value is small and outliers are few. More specifically, by modeling the problem as $y = \mathsf{A}x+\xi$, where $x\in \mathbf{R}^n$ is the vector containing the weights contributed by each dictionary entry, we assume that the distribution of *x* is Gaussian, $x \sim \mathrm{N}(0,\theta)$, where the variance θ is a random variable distributed according to the gamma distribution. Computing the MAP estimate of the posterior density amounts to solving the linear least squares problem: $\min_x\left\{\frac{1}{2\sigma^2}\|y-\mathsf{A}x\|_2^2+\frac{1}{2}\|\mathsf{T}_\theta^{-1/2}x\|_2^2\right\}$, where T_θ is a diagonal matrix containing the variances $\theta_j,\ j=1,\ldots,n,$ of the weights assigned to each dictionary entry. The minimization problem is solved iteratively, alternating between updating *x* and θ, and at the k^{th} iteration, the entries corresponding to the lowest values of the current solution $x^{(k)}$ are discarded and the next iteration begins with a smaller dictionary, pruning the dictionary at each step. Fractions are computed by grouping the remaining dictionary entries based on correlation and then applying the pseudoinverse model. [2]

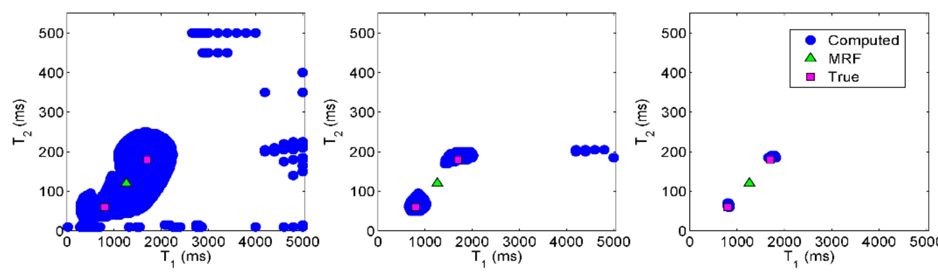

Figure 2: T_1, T_2 scatter plots of remaining dictionary entries after pruning at iterations 8 (left), 20 (middle) and 34 (right). The true T_1, T_2 pairs are shown in pink squares, the results from MRF template matching is displayed as a green triangle.

Results: To assess the viability of this model, we design a mixed signal from two MRF-FISP [4] dictionary entries, combined in the fractions shown in Table 1. We retrospectively simulate aliasing using a one-shot spiral trajectory and add Gaussian noise to the normalized signal; the true and simulated signals are shown in Fig. 1. Scatter plots showing the T_1, T_2 combinations from the dictionary that remain after pruning are shown at iterations 8, 20, and 34 in Fig. 2. This signal is passed through the algorithm for 42 iterations, reducing the dictionary by a factor of 1/6 at each iteration. At iteration 42 we are left with the 5 entries shown in Table 2.

T_1	T_2	Fraction
820	60	0.4371
840	65	
1760	185	0.5629
1780	185	
1800	190	

Table 2: Remaining dictionary entries and their weightings at iteration 42.

Discussion: In this example, we apply a Bayesian statistical framework to determine the tissue types present in a mixed MRF signal. By solving the minimization problem with a mixed signal *y*, we prune the dictionary down to five entries, close to the original components that comprise the mixed signal. The important thing to note is that no assumptions were made *a priori* as to which dictionary entries should make up the solution, only assumptions about the distribution of the entries was made, allowing the algorithm to fit a solution to the data without forcing any one particular solution. These preliminary results are promising, though there are hurdles that will need to be overcome to apply the model to real data, including how to choose the optimal parameters for the minimization problem.

Acknowledgements: The authors would like to acknowledge funding from Siemens Healthcare and NIH grants 1R01EB016728-01A1 and 5R01EB017219-02.

References: 1. Ma D, et al. Nature 495, 187-92 (2013). 2. Deshmane, et al., Proc. ISMRM 22, 94 (2014). 3. Calvetti D, Somersalo E., Inverse Problems 24, 034013. 4. Jiang, et al., Proc. ISMRM 22, 4290 (2014).