Nonlinear Dimensionality Reduction for Magnetic Resonance Fingerprinting with Application to Partial Volume

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Target Audience: Scientists interested in dimensionality reduction and quantitative MRI, including the partial volume effect.
Purpose: In Magnetic Resonance Fingerprinting (MRF)[1], quantitative maps of tissue parameters are produced by matching acquired MR signals to a precomputed dictionary, which is formed using Bloch simulation. By varying the MR parameters such as the flip angle and repetition times, a unique time course is generated for each parameter combination. Previously, it was demonstrated that linear compression in the time domain is possible using singular value decomposition (SVD) to compress the dictionary entries to less than 20% of the original length without sacrificing accuracy of the resulting parameter maps.[2] However, since each dictionary entry is uniquely defined by two inputs, namely T_1 and T_2, we believe that further compression is possible, resulting in new potentials for MRF calculations. Here we propose to compress the dictionary in a nonlinear manner so that each entry is uniquely represented by a point on a manifold in $\mathbf{R}^3$. To this end, we apply kernel principal component analysis (KPCA) [3] to achieve the nonlinear dimensionality reduction to $\mathbf{R}^3$ and then explore the application of this reduced subspace model to the partial volume problem.

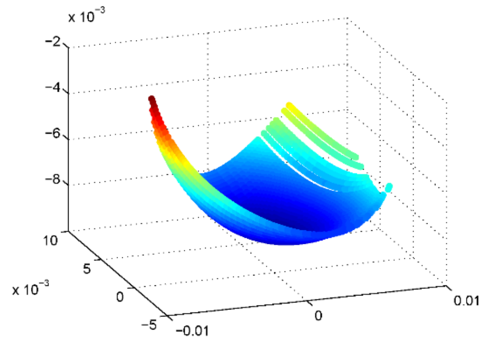

Figure 1: A 3D MRF-FISP dictionary manifold as computed using KPCA for dimensionality reduction.

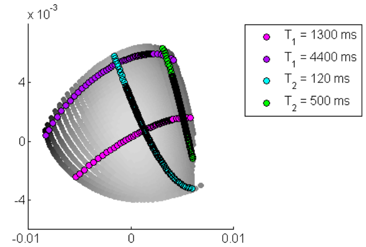

Figure 2: The same manifold as in Figure 1, but projected onto the first two KPCA components. Four curves on the manifold are labeled, corresponding to specific values for T_1 and T_2 as shown in the legend.

Methods: Consider the MRF-FISP [4] dictionary as an n x t complex-valued matrix, where n is the number of dictionary entries, or T_1, T_2 combinations, and t is the number of time points. In this particular example, n = 9820 and t = 2500. To compress the dimension t, we apply KPCA using a Gaussian kernel with target dimensionality of 3, accounting for T_1, T_2, and a free parameter. The compressed dictionary, or manifold, is shown in Fig. 1, and a two-dimensional projection is shown in Fig. 2. To simulate the partial volume problem, two dictionary entries, denoted tissues A and B, are combined in various ratios and then projected into $\mathbf{R}^3$ using the KPCA kernel and eigenvectors previously computed in the dictionary compression. To recover the ratios, we compute the distances between the mixed signals and their pure dictionary components on the manifold. In particular, we expect that the mixed signals should lie along a linear continuum between the pure tissues A and B on the manifold. To achieve this, the mixed signals are projected onto the line between tissues A and B and the ratios are computed using these distances. A fifth degree polynomial approximation of the manifold is shown in Fig. 3 along with the compressed mixed signals in round dots. The line on to which the mixed signals are to be projected is shown in black.

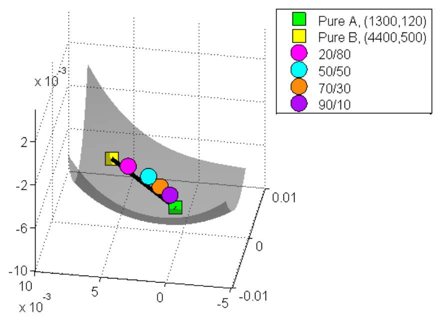

Figure 3: A polynomial approximation to the dictionary manifold and the projected pure (square markers) and mixed (round markers) signals.

Results: Two dictionary entries from the FISP sequence were chosen, tissue A with T_1 = 1300 ms and T_2 = 120 ms and tissue B with T_1 = 4400 ms and T_2 = 500 ms. The entries were combined in the ratios shown in the first column of Table 1 and then aliasing was simulated using a one-shot spiral trajectory. Gaussian noise was added to the normalized signal. We are able to recover the tissue ratios within 5 percentage points of the true ratios, as shown in the two columns of Table 1 labeled "Predicted," without performing a matrix inversion at each pixel.

True	Predicted	
A/B	A	B
.20/.80	.2279	.7721
.50/.50	.5076	.4924
.70/.30	.6982	.3018
.90/.10	.8600	.1400

Table 1: True and predicted signal ratios.

Discussion: Due to the nature of the Bloch simulation in MRF, we expect that the dictionary, a complex matrix of size n x t, should be compressible to a real matrix of size n x 3, accounting for T_1, T_2, and a free parameter. This compressibility is clearly seen with MRF-FISP using KPCA, as shown in Fig. 1. A promising application to the partial volume problem is also presented. As is the case in previously presented works on MRF and partial volume, [5] the tissue types that make up the mixed signals must be known *a priori*, though in future work, we anticipate building this uncertainty into our model.

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References: 1. Ma D, et al. Nature 495, 187-92 (2013). 2. McGivney D, et al. IEEE TMI, 10.1109/TMI.2014.2337321 (2014) 3. Schölkopf B, et al. Kernel Principal Component Analysis, in *Artificial Neural Networks – ICANN'97*, 1997. 4. Jiang, et al., Proc. ISMRM 22, 4290 (2014).
5. Deshmane, et al., Proc. ISMRM 22, 94 (2014).