

QUANTITATIVE SUSCEPTIBILITY MAPPING BY SPATIAL LAPLACE REGULARIZATION

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Purpose: Quantitative susceptibility mapping (QSM) provides a novel contrast mechanism in MRI. Many QSM methods have been proposed to solve the magnetic field to susceptibility source inverse problem, including thresholded k-space division (TKD), morphology enabled dipole inversion (MEDI) and calculation of susceptibility through multiple orientation sampling (COSMOS) ¹⁻⁴. Most of the studies adopt dipole kernel in the k-space to convert unwrapped phase image into susceptibility image. Phase unwrapping process and Fourier transform are required in these methods. Since phase unwrapping process may induce errors at the regions of rapid phase changing or low signal-noise-ratio, it is desirable to design a QSM algorithm that avoids phase unwrapping necessity. As for k-space calculation, due to the limited sampled window, multiplication at boundary truncation leads to ripple artifact in spatial domain. In this study, we propose a QSM method by calculating susceptibility in spatial domain directly from Laplace filtered phase data. Preliminary results on simulation and human brain data indicate the advantage of this Laplace regularization QSM.

Methods: If the magnetic field is along z-direction, the relation between unwrapped phase and magnetic susceptibility is given by $\nabla^2 \varphi(x, y, z) = \gamma H_0 TE (d^2 / dx^2 + d^2 / dy^2 - 2d^2 / dz^2) \chi(x, y, z)$, where φ is MRI phase, γ is the gyromagnetic ratio, H_0 is magnetic field strength, TE is time of echo and χ is magnetic susceptibility. The derivative operation on the right side is performed in the spatial domain according the following equation: $d^2 f(\xi) / d\xi^2 = f(\xi + 1) + f(\xi - 1) - 2f(\xi)$. As a result, the operation $(d^2 / dx^2 + d^2 / dy^2 - 2d^2 / dz^2)$ is equal to multiplying a matrix A. The Laplace operator on the left side is calculated from the sine and cosine of wrapped phase image based on the following equation to avoid the phase unwrapping process: $\nabla^2 \varphi = \cos \varphi \cdot \nabla^2(\sin \varphi) - \sin \varphi \cdot \nabla^2(\cos \varphi)$ ⁴. Therefore, the relation between susceptibility and unwrapped phase can be rewritten into a problem of linear matrix multiplication $AX = b$. The problem can be solved as a minimization problem by using regularization terms as follow: $\min_x \|W(Ax - b)\|_2^2 + \alpha \|W_0 X\|_2^2 + \beta \|W_1 G X\|_2^2$. In the first regularization term, W equals 1 at region of interest as a mask; In the second regularization term W_0 equals 1

at a region where the susceptibility is known a priori to be homogeneous to enforce the uniform zero solution; In the third regularization term, W_1 is the inverse of the magnitude image gradient along three directions as susceptibility gradient regularization. The parameters α and β are used to control the weight of each regularization term². The minimization problem can be solved by conjugate gradient algorithm. Simulation was performed by assigning 9 spherical areas with different magnetic susceptibility. Magnetic field was generated from the predefined magnetic susceptibility distribution. Healthy volunteers were scanned by a Philips 3T system (Philips Healthcare, Best, The Netherlands) with multi-slice gradient echo pulse sequence with following configuration: FOV=200×224×80 mm³; voxel size=1×1×2 mm³; TR=50 ms; TE=14 ms. Susceptibility maps were calculated by both TKD method and proposed Laplace regularization method. Post-processing algorithms were implemented in MATLAB (v. R2010a, The Mathworks, Natick, MA).

Results: Fig. 1 shows the simulation results of nine spherical regions with different susceptibility. Fig. 1c shows the QSM result calculated by TKD method at threshold=0.2 to avoid the singularity points. Fig. 1d shows the QSM result calculated by proposed Laplace regularization method without mask and gradient regularization ($\alpha=\beta=0$). Fig. 1e plots the relation between calculated susceptibility and reference susceptibility. Two lines are separated by adding a constant value 5 to the TKD calculated susceptibility. The error bar shows that the root-mean-square-

error of TKD method at threshold=0.2 is larger than Laplace regularization method especially at large susceptibility region. Fig. 2 shows the result of human brain. Fig. 2e and Fig. 2f are unwrapped phase images obtained by PRELUDE algorithm⁵. Susceptibility maps Fig. 2g and Fig. 2h are calculated by TKD method with threshold=0.2. Susceptibility maps Fig. 2i and Fig. 2j are calculated by Laplace regularization method from wrapped images Fig. 2c and Fig. 2d with gradient regularization from magnitude image Fig. 2a and Fig. 2b.

Discussion: In this study, we use Laplace filtered phase data directly for QSM calculation and all the calculation is in the spatial domain. As Laplace filter is functioned as a high-pass filter, Laplace regularization method may avoid the requirement of high-pass filtering that is applied in most QSM algorithms. Comparison between Fig. 1c and Fig. 1d shows that Laplace regularization method may induce less streak artifacts than k-space inversion method due to thresholded truncation. From the comparison between Fig. 2g, h and Fig. 2i, j, we may notice that the susceptibility maps from TKD and Laplace regularization method show similar susceptibility distribution. Fig. 2i,j show relative less blurring effect than that of Fig. 2g,h, which is probably due to thresholded truncation or Fourier associated k-space truncation in TKD.

Conclusion: In this study, we propose a QSM method by directly using Laplace filtered phase data in spatial domain.

Comparison between TKD method and Laplace regularization in both simulation and human brain experimental data validate the performance of this algorithm.

References: [1] Shmueli K, et al. Magn Reson Med, 2009, 62(6), 1510-1522; [2] Rochefort L, et al. Magn Reson Med, 2010,63(1), 194-206; [3] Liu T, et al. Magn Reson Med, 2009, 61(1), 196-204. [4] Scheweser F, et al, Magn Reson Med, 2013, 69(6), 1582-1594. [5] Smith S, et al, NeuroImage, 2004;23:208-219.

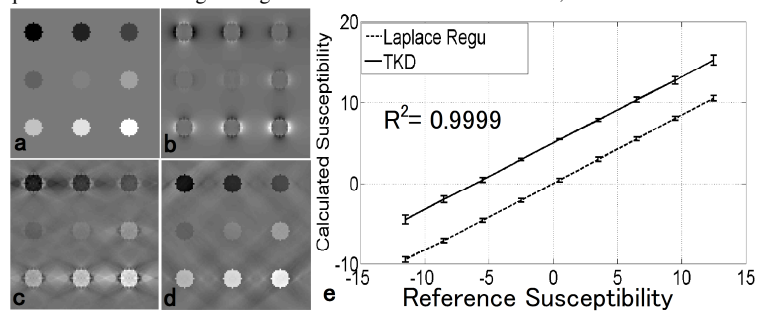


Fig.1: Simulation results. (a) predefined magnetic susceptibility distribution; (b) magnetic field map generated from predefined susceptibility; (c) QSM results calculated by TKD method; (d) QSM result calculated by Laplace regularization method; (e) Plot of calculated susceptibility with reference susceptibility (TKD calculated susceptibility is add 5 for lines separation).

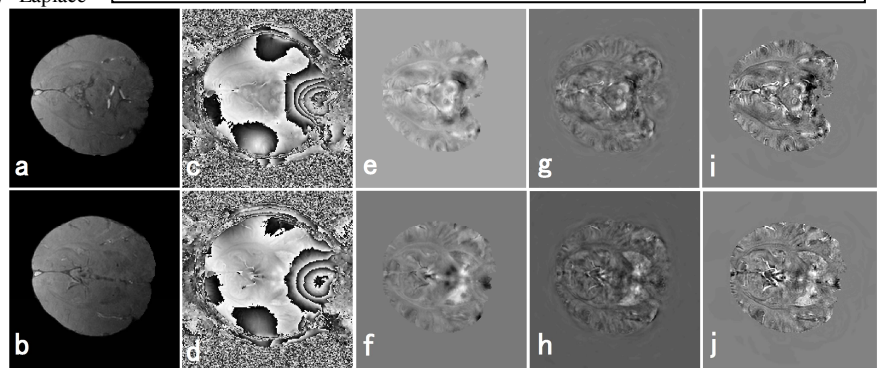


Fig.2: Experimental human brain results. (a,b) magnitude images; (c,d) wrapped phase image; (e,f) unwrapped phase image; (g,h) QSM calculated by TKD method from unwrapped phase data; (i,j) QSM calculated by Laplace regularization method from wrapped phase data.