

A simple point-wise formula for double excitation MREPT suitable for reconstructing boundaries

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Introduction: MREPT for reconstructing conductivity (σ) and permittivity (ϵ) of tissues has been proposed by Haacke *et al*¹ and various improvements have so far been proposed and implemented^{2,3}. Apart from various assumptions made for deriving easily implementable algorithms, one of the major problem is still reconstructing σ and ϵ at the tissue boundaries where gradients of these variables are high. Recently, Sodickson *et al* have proposed algorithms which are not easy to apply in practice⁴. We have recently proposed a convection-reaction equation based MREPT (cr-MREPT) method in order to solve this problem^{5,6}. In this method single or double excitation data are used to reconstruct sigma and epsilon not only where they vary slowly but also in the boundary (or transition) regions. We have solved the problem using a FEM-like triangular mesh based method whereby the whole domain is reconstructed at once by solving a matrix equation. However it is preferable at times to obtain a point-wise (or local) formula in order to reconstruct only a region of interest. Point-wise formula is prone to noise effects and therefore some regularization or filtering may be necessary. **Theory:** We had previously derived

the equation $\mathbf{F} \cdot \nabla u + \nabla^2 H^+ u = i\omega\mu H^+$ where $u = 1/(\sigma + i\omega\epsilon)$, and

$$\mathbf{F} = \left[\frac{\partial H^+}{\partial x} - i \frac{\partial H^+}{\partial y}, i \frac{\partial H^+}{\partial x} + \frac{\partial H^+}{\partial y}, \frac{\partial H^+}{\partial z} \right]^T. \text{ By assuming } \frac{\partial H^+}{\partial z} = 0$$

and $\frac{\partial u}{\partial z} = 0$ one can obtain the 2D version of this equation. Defining

$$\alpha = \frac{\partial H^+}{\partial x} - i \frac{\partial H^+}{\partial y}, \beta = \nabla^2 H^+, \gamma = i\omega\mu H^+, \text{ and } v = \frac{\partial u}{\partial x} + i \frac{\partial u}{\partial y} \text{ we}$$

obtain $\alpha v + \beta u = \gamma$. If we have two excitations then we have the two equation

set $\alpha_1 v + \beta_1 u = \gamma_1$ and $\alpha_2 v + \beta_2 u = \gamma_2$ from which we can solve for u at each spatial point as

$$u = (\alpha_1 \gamma_2 - \alpha_2 \gamma_1) / (\alpha_1 \beta_2 - \alpha_2 \beta_1). \text{ In this case, } \sigma = \text{real}\{(\alpha_1 \beta_2 - \alpha_2 \beta_1) / (\alpha_1 \gamma_2 - \alpha_2 \gamma_1)\}$$

and $\epsilon = \text{imag}\{(\alpha_1 \beta_2 - \alpha_2 \beta_1) / \omega(\alpha_1 \gamma_2 - \alpha_2 \gamma_1)\}$. For $n > 2$ excitations the set of n equations may be

solved in the least squares sense. **Methods:** In order to test the algorithm we have generated simulated data, i.e. H^+ , using Comsol Multiphysics (Comsol AB, Sweden). A birdcage body coil for transmit and receive is modeled⁷. The cylindrical simulation phantom with radius 7.2 cm and height 20 cm is placed along the axis of the birdcage coil. The z-independent electrical conductivity distribution at the central transversal slice is shown in Fig 3(a). The conductivity transition is not sharp but it is tapered. It is assumed that two excitations are realized. In one case, the whole cylinder is placed in the birdcage coil and in the second case, part A of the cylinder (shown in Fig. 1) is cut away, i.e. this part is assigned the same electrical properties as for air. The region of interest is the combinations of part B, C₁ and C₂ and the overall effect is to have two different H^+ distributions in the ROI.

Results: The α , β and γ distributions for the two different excitation cases are shown in Fig. 2. It is observed that the regions where the convective field (α) is low, named low convective field (LCF) region, do not overlap for the excitations. This is in fact what is aimed from using two excitations since if both are close to zero then σ and ϵ are poorly defined. The conductivity distribution reconstructed using the proposed method is shown in Fig 3(b). For the first excitation, the conductivity reconstruction obtained by Haacke's formula ($u = \gamma/\beta$), which neglects the α term, is shown in Fig 3(c). It is clear that Haacke's formula gives erroneous results in the transition regions whereas the proposed point-wise double-excitation method reconstructs the conductivity in the transition regions satisfactorily. The performance of the algorithm against noise is also analyzed. Gaussian distributed white noise (std. dev is 1 nT) is added to H^+ distributions of both excitations. Since $\nabla^2 H^+$ calculations are very sensitive to noise effects, the noise added H^+ distributions are low-pass filtered with a spatial cut-off frequency of 711 m⁻¹ before reconstruction. The result of the electrical conductivity reconstruction is shown in Fig. 3(d). It is observed that despite the presence of noise, the conductivity reconstruction at the transition region is still successful.

Discussion and Conclusion: The advantage of the double-excitation point-wise formula is that it reconstructs the conductivity at the transition region as well. In practice, the second excitation may be obtained by attaching pads of appropriate EPs⁵ to the surface of the imaging subject. Also, two excitations may be realized by multi-coil transmit and receive configuration similar to Sodickson *et al*⁴. In another paper submitted to this conference by our group, we suggest a multi-coil configuration based on birdcage body coil transmit and phased-array head coil receive⁸. Improvements of the algorithm by locally averaged version and testing against experimental data are on the way.

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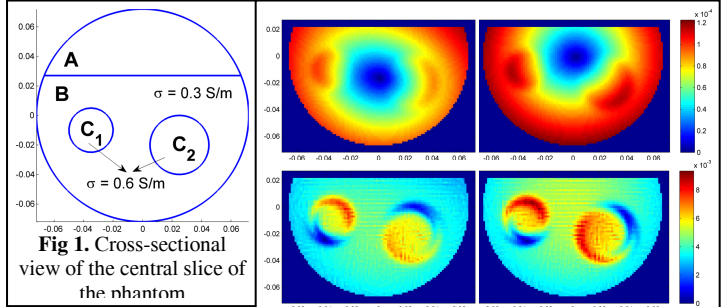


Fig 1. Cross-sectional view of the central slice of the phantom

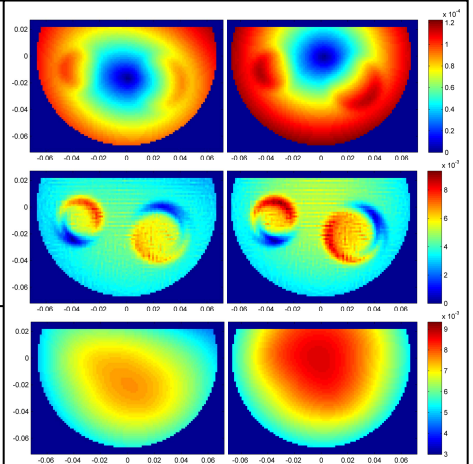


Fig 2. α , β and γ distributions. Left and right column correspond to first and second excitations.

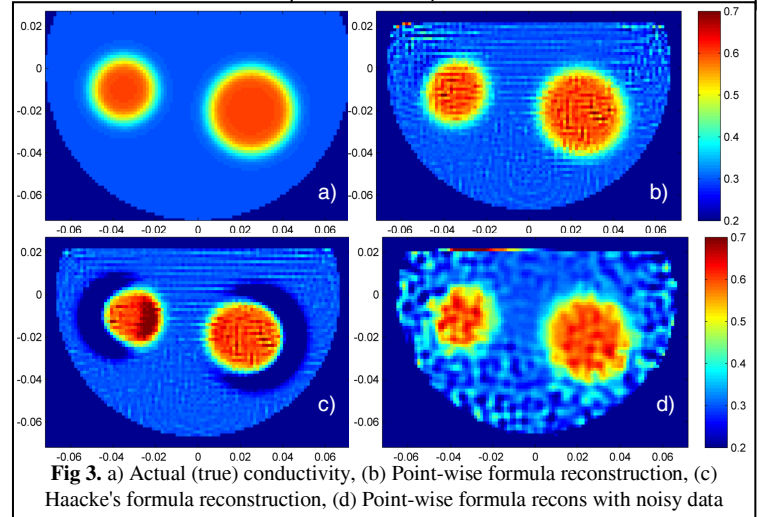


Fig 3. a) Actual (true) conductivity, (b) Point-wise formula reconstruction, (c) Haacke's formula reconstruction, (d) Point-wise formula recons with noisy data