Simultaneous Determination of Electrical Properties and Proton Density in a Generalized Gradient-based Electrical Properties Tomography

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Purpose The original electrical properties tomography (EPT) was proposed based on a homogeneous Helmholtz equation to extract conductivity σ and permittivity ε (EP) of tissue noninvasively using MRI[1,2]. However, near boundaries separating different electrical properties, this simplified formula does not hold anymore, resulting in significant boundary artifacts[3]. In a previous study[4], we have proposed a gradient-based EPT algorithm (gEPT), in which it was shown that the gradient **g** of EP can be obtained to provide EP maps with significantly improved boundary reconstruction and robustness against measurement noise. In this abstract, we introduce a more generalized gEPT framework to be able to derive **g** under realistic situations when the measured receive B1 field is biased by the unknown proton density (ρ). Our results show that both EP and proton density ρ can be reliably estimated using this proposed approach.

Theory Based on the time-harmonic Maxwell's equations, ignoring the unknown B_z component, inhomogeneous Helmholtz equations in the form of transmit (B_1^+) or receive (B_1^-) B1 fields can be expressed as Eqs. 1 or 2[4,5], respectively, in which the complex permittivity $\varepsilon_c=\varepsilon-i\sigma/\omega$, ω the Larmor angular frequency, μ_0 magnetic permeability of free space, gradient $\mathbf{g}=\nabla\ln\varepsilon_c$, and $\hat{B}_1^-$ denotes the measurable proton-density-weighted receive B1 (ρB_1^-) with $\gamma=1/\rho$. Equations 1 and 2 can further be extended into the magnitude $\left|B_1^+\right|$, $\left|\hat{B}_1^-\right|$ and phase $\phi^\pm$ components of B_1^+ and B_1^- fields. Using a multi-channel transmit/receive array RF coil, $\left|B_{1j}^+\right|$, $\left|\hat{B}_{1k}^-\right|$, *relative* phase difference $\Delta\phi_j^+=\phi_j^+-\phi_n^+$ and *transceiver* phase $\phi_{nk}=\phi_n^++\phi_k^-$ can be measured[6], where j and k denote the index of transmit and receive channels, respectively, and n a reference transmit channel. Knowing those information is sufficient to derive the unknown **g**, $\nabla\ln\gamma$, $\nabla\phi_n^+$, σ and ε based on Eqs. 1 and 2. Once **g** and $\nabla\ln\gamma$ are obtained, maps of σ, ε and ρ can be reconstructed from the gradients using the finite difference method.

$$\nabla^2 B_1^+ = -\omega^2\mu_0 B_1^+\varepsilon_c + (\nabla B_1^+)^T \begin{bmatrix}1 & i & 0\\ -i & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\mathbf{g} \quad (1)$$

$$\nabla^2\left(\gamma\hat{B}_1^-\right) = -\omega^2\mu_0\gamma\hat{B}_1^-\varepsilon_c + \nabla\left(\gamma\hat{B}_1^-\right)^T \begin{bmatrix}1 & -i & 0\\ i & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\mathbf{g} \quad (2)$$

Methods ***Simulation*** The proposed generalized gEPT algorithm was first evaluated utilizing finite-difference time-domain (FDTD) simulation. A sixteen-channel RF microstrip array coil[7] was rebuilt in the simulation software SEMCAD and loaded with an anatomically realistic human head model (Duke). Sixteen channels of B_1^+ and B_1^- were computed with a spatial resolution of 2x2x2 mm^3. ***Experiment*** To validate the proposed method, a saline-gel phantom was built, including three different components of various concentration of NaCl. The corresponding conductivity and permittivity of the three solutions at 298MHz were measured using a dielectric probe as shown in Table 1. MRI experiment was performed on a 7T Siemens MAGNETOM scanner, equipped with 16-channel RF power amplifier and 32-channel receiver. The sixteen-channel RF microstrip array coil, modeled in the simulation study, was utilized to transmit RF energy and receive MR signal. For each channel, $\left|B_1^+\right|$ and $\left|\hat{B}_1^-\right|$ were obtained using a hybrid B1 mapping technique[6,8]. Relative phase $\Delta\phi_j^+$ and transceiver phase ϕ_{jk} were obtained from the signal phase of large flip angle gradient echo recalled (GRE) sequences. A ΔB_0 map was measured with GRE phase maps of two different echo times, to correct ϕ_{jk} from inhomogeneous B_0.

Results The reconstructed maps of conductivity σ, permittivity ε and proton density ρ of the simulation study are shown in Fig. 1 in comparison with corresponding target maps of the model input. Reconstructed relative errors (RE) and correlation coefficients (CC) are RE_σ=9.2%, RE_ε=9.8%, RE_ρ=4.5%, CC_σ=0.98, CC_ε=0.85 and CC_ρ=0.97. Results of reconstructed σ, ε and ρ of the phantom experiment are summarized in Table 1. Target ρ throughout the phantom is not directly measurable but assumed to be uniform (or normalized to be 1) because only NaCl concentration was slightly adjusted in the recipes. As we can see, using the proposed generalized gEPT approach, not only the electrical properties but also unknown ρ can be faithfully derived from the measurable B1 field information. We do notice a significant drop of the reconstructed σ of phantom component #3. This could be due to errors of numerical differentiation near the vicinity of strong contrast of σ between #3 and #1. Finer imaging resolution may help improve the numerical performance.

Table 1 Results of the phantom experiment. Units: σ – [Sm^{-1}]; ε – [ε_0]; ρ – [a.u.].

	#1			#2			#3		
	σ	ε	ρ	σ	ε	ρ	σ	ε	ρ
Target	0.12	78	--	0.34	77	--	1.5	77	--
Recon.	0.19	76	1.00	0.34	73	0.97	0.94	72	0.94
±Std.	±0.16	±5	±0.07	±0.06	±3	±0.03	±0.21	±2	±0.06

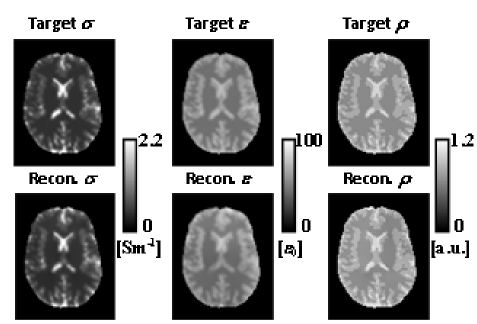

Figure 1 Results of simulation

Discussion and Conclusion In this study, we extended the generality of the previously proposed gEPT approach to quantitatively calculate gradient **g** and unknown proton density ρ at the same time. The method is generally applicable to any MR system with a multi-channel transmit/receive RF coil. Future work will be focused on advanced algorithms to combine the gradient **g** and absolute σ and ε; the latter can also be derived in the solution but are more sensitive to noise.

Reference [1] Wen, *Proc. SPIE* 2003, 5030:471; [2] Katscher *et al., IEEE Trans Med Imaging* 2009, 28:1365; [3] Seo et al., *IEEE Trans Med Imaging* 2012, 31:430; [4] Liu *et al.*, *ISMRM* 2013, 3486; [5] Zhang et al., *IEEE Trans Med Imaging* 2013, 32:1058; [6] Van de Moortele *et al.*, *ISMRM* 2007, 1676; [7] Adriany *et al. MRM* 2008, 59:590; [8] Van de Moortele *et al.*, *ISMRM* 2009, 367 **Acknowledgement** NIH R21 EB017069-01A1, R01 EB006433, R01 EB007920, R21 EB014353, R21 EB009138, T32 EB008389, P41 EB015894, 2R01 EB006835, 2R01 EB007327, S10 RR26783 and WM KECK Foundation.